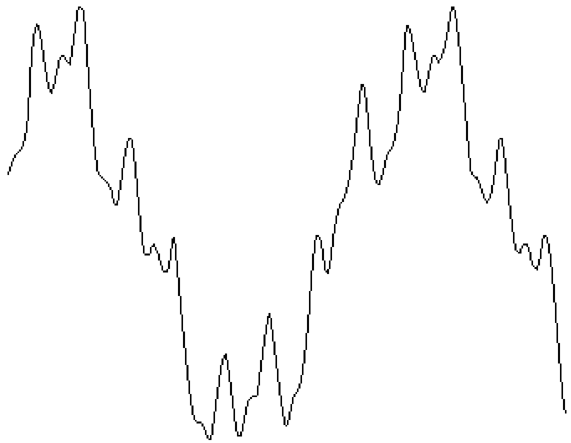
Digital Image Processing

Chapter 4:

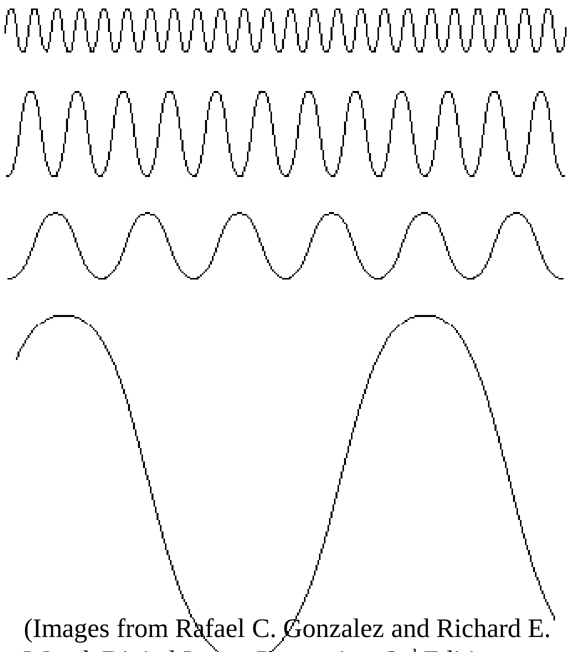
Image Enhancement in the Frequency Domain

Background: Fourier Series



Fourier series:

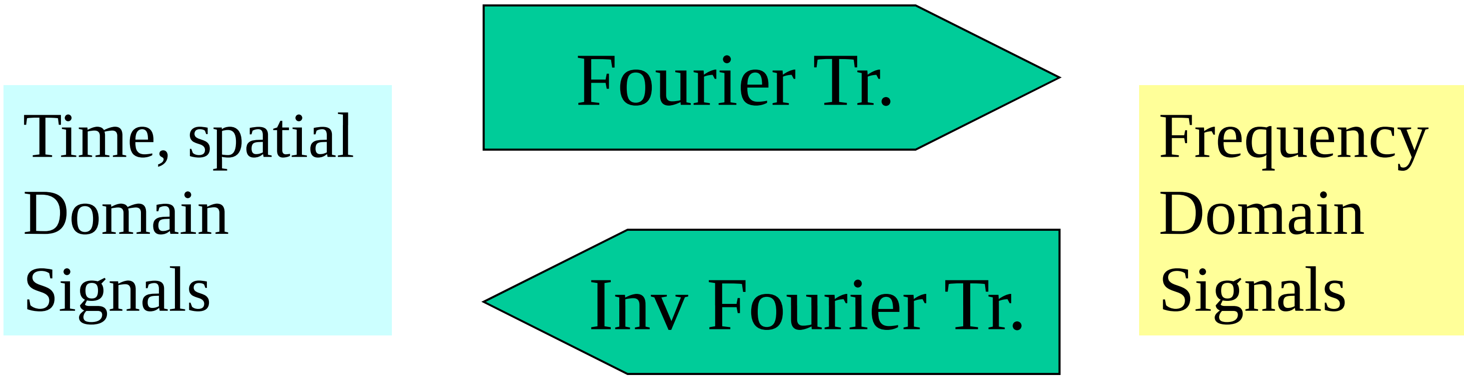
Any periodic signals can be viewed as weighted sum of sinusoidal signals with different frequencies



Frequency Domain: view frequency as an independent variable

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Fourier Tr. and Frequency Domain



1-D, Continuous case

Fourier Tr.:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

Inv. Fourier Tr.:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

Fourier Tr. and Frequency Domain (cont.)

1-D, Discrete case

Fourier Tr.: $$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M} \qquad u = 0,\ldots,M-1$$

Inv. Fourier Tr.: $$f(x) = \sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M} \qquad x = 0,\ldots,M-1$$

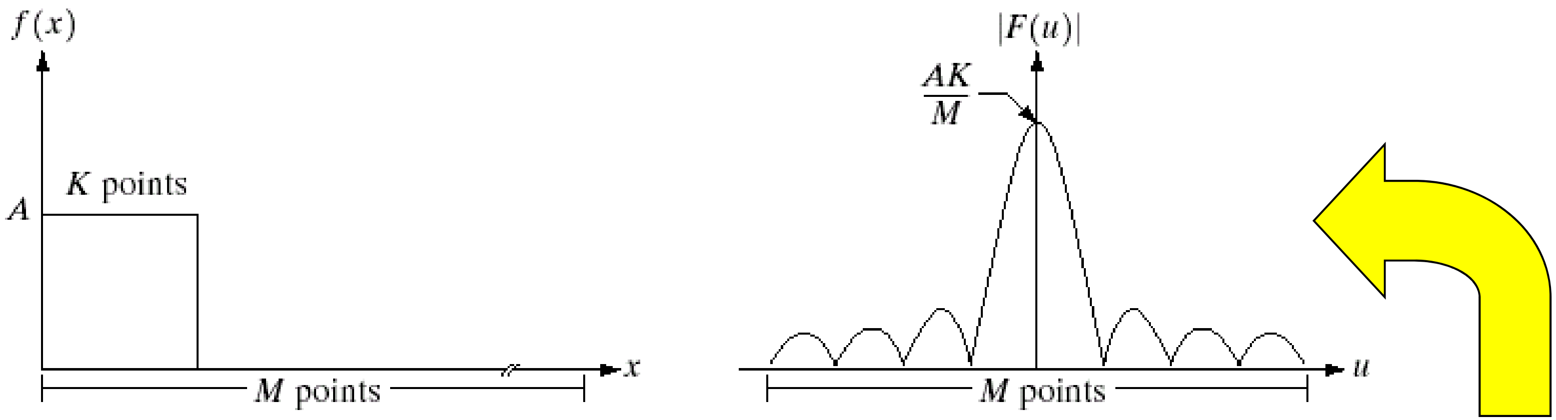
$F(u)$ can be written as

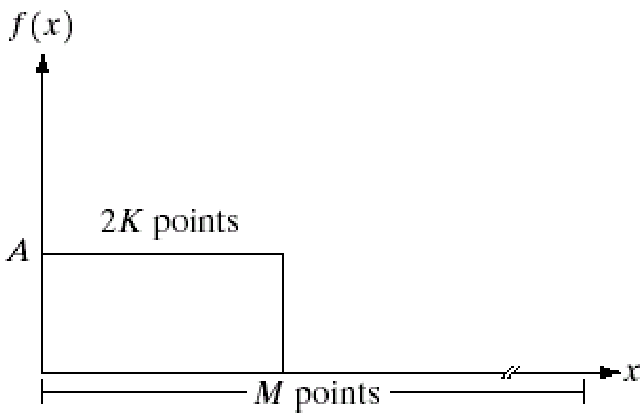
$$F(u) = R(u) + jI(u) \quad \text{or} \quad F(u) = |F(u)|e^{-j\phi(u)}$$

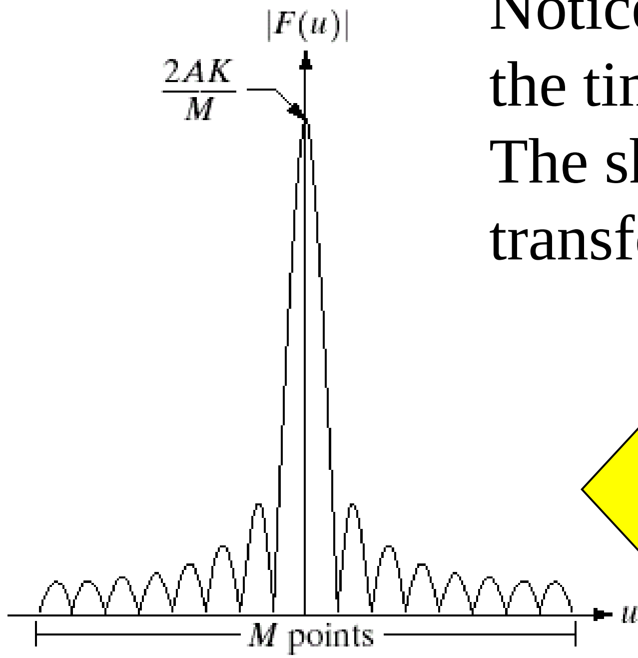
where

$$|F(u)| = \sqrt{R(u)^2 + I(u)^2} \qquad \phi(u) = \tan^{-1}\left(\frac{I(u)}{R(u)}\right)$$

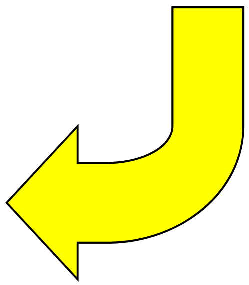
Example of 1-D Fourier Transforms







Notice that the longer the time domain signal, The shorter its Fourier transform



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Relation Between Δx and Δu

For a signal $f(x)$ with M points, let spatial resolution Δx be space between samples in $f(x)$ and let frequency resolution Δu be space between frequencies components in $F(u)$, we have

$$\Delta u = \frac{1}{M \Delta x}$$

Example: for a signal f(x) with sampling period 0.5 sec, 100 point, we will get frequency resolution equal to

$$\Delta u = \frac{1}{100 \times 0.5} = 0.02 \ \ \text{Hz}$$

This means that in $F(u)$ we can distinguish 2 frequencies that are apart by 0.02 Hertz or more.

2-Dimensional Discrete Fourier Transform

For an image of size MxN pixels

2-D DFT

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

u = frequency in x direction, $u = 0, \ldots, M-1$
v = frequency in y direction, $v = 0, \ldots, N-1$

2-D IDFT

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)}$$

$x = 0, \ldots, M-1$
$y = 0, \ldots, N-1$

2-Dimensional Discrete Fourier Transform (cont.)

$F(u,v)$ can be written as

$$F(u,v) = R(u,v) + jI(u,v) \quad \text{or} \quad F(u,v) = |F(u,v)|e^{-j\phi(u,v)}$$

where

$$|F(u,v)| = \sqrt{R(u,v)^2 + I(u,v)^2} \qquad \phi(u,v) = \tan^{-1}\left(\frac{I(u,v)}{R(u,v)}\right)$$

For the purpose of viewing, we usually display only the Magnitude part of $F(u,v)$

2-D DFT Properties

TABLE 4.1
Summary of some important properties of the 2-D Fourier transform.

Property	Expression(s)
Fourier transform	$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$
Inverse Fourier transform	$f(x,y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$
Polar representation	$F(u,v) = \lvert F(u,v)\rvert e^{-j\phi(u,v)}$
Spectrum	$\lvert F(u,v)\rvert = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$, $R = \text{Real}(F)$ and $I = \text{Imag}(F)$
Phase angle	$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$
Power spectrum	$P(u,v) = \lvert F(u,v)\rvert^2$
Average value	$\bar{f}(x,y) = F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$
Translation	$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$ $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M+vy_0/N)}$ When $x_0 = u_0 = M/2$ and $y_0 = v_0 = N/2$, then $f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2, v-N/2)$ $f(x-M/2, y-N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

2-D DFT Properties (cont.)

TABLE 4.1 *(continued)*

Conjugate symmetry	$F(u, v) = F^*(-u, -v)$ $\lvert F(u, v)\rvert = \lvert F(-u, -v)\rvert$
Differentiation	$\dfrac{\partial^n f(x, y)}{\partial x^n} \Leftrightarrow (ju)^n F(u, v)$ $(-jx)^n f(x, y) \Leftrightarrow \dfrac{\partial^n F(u, v)}{\partial u^n}$
Laplacian	$\nabla^2 f(x, y) \Leftrightarrow -(u^2 + v^2)F(u, v)$
Distributivity	$\Im[f_1(x, y) + f_2(x, y)] = \Im[f_1(x, y)] + \Im[f_2(x, y)]$ $\Im[f_1(x, y) \cdot f_2(x, y)] \neq \Im[f_1(x, y)] \cdot \Im[f_2(x, y)]$
Scaling	$af(x, y) \Leftrightarrow aF(u, v), f(ax, by) \Leftrightarrow \dfrac{1}{\lvert ab\rvert} F(u/a, v/b)$
Rotation	$x = r\cos\theta \quad y = r\sin\theta \quad u = \omega\cos\varphi \quad v = \omega\sin\varphi$ $f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$
Periodicity	$F(u, v) = F(u + M, v) = F(u, v + N) = F(u + M, v + N)$ $f(x, y) = f(x + M, y) = f(x, y + N) = f(x + M, y + N)$
Separability	See Eqs. (4.6-14) and (4.6-15). Separability implies that we can compute the 2-D transform of an image by first computing 1-D transforms along each row of the image, and then computing a 1-D transform along each column of this intermediate result. The reverse, columns and then rows, yields the same result.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

2-D DFT Properties (cont.)

Property	Expression(s)
Computation of the inverse Fourier transform using a forward transform algorithm	$\frac{1}{MN} f^*(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F^*(u, v) e^{-j2\pi(ux/M + vy/N)}$ This equation indicates that inputting the function $F^*(u$, [illegible] into an algorithm designed to compute the forward tra[illegible] (right side of the preceding equation) yields $f^*(x, y)/MN$. Taking the complex conjugate and multiplying this result by MN gives the desired inverse.
Convolution†	$f(x, y) * h(x, y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) h(x - m, y - n)$
Correlation†	$f(x, y) \circ h(x, y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f^*(m, n) h(x + m, y + n)$
Convolution theorem†	$f(x, y) * h(x, y) \Leftrightarrow F(u, v) H(u, v)$; $f(x, y) h(x, y) \Leftrightarrow F(u, v) * H(u, v)$
Correlation theorem†	$f(x, y) \circ h(x, y) \Leftrightarrow F^*(u, v) H(u, v)$; $f^*(x, y) h(x, y) \Leftrightarrow F(u, v) \circ H(u, v)$

TABLE 4.1 *(continued)*

2-D DFT Properties (cont.)

TABLE 4.1
(continued)

Some useful FT pairs:

Impulse	$\delta(x, y) \Leftrightarrow 1$
Gaussian	$A\sqrt{2\pi}\sigma e^{-2\pi^2\sigma^2(x^2+y^2)} \Leftrightarrow Ae^{-(u^2+v^2)/2\sigma^2}$
Rectangle	$\text{rect}[a, b] \Leftrightarrow ab\frac{\sin(\pi ua)}{(\pi ua)}\frac{\sin(\pi vb)}{(\pi vb)}e^{-j\pi(ua+vb)}$
Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow \frac{1}{2}[\delta(u + u_0, v + v_0) + \delta(u - u_0, v - v_0)]$
Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow j\frac{1}{2}[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)]$

† Assumes that functions have been extended by zero padding.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Computational Advantage of FFT Compared to DFT

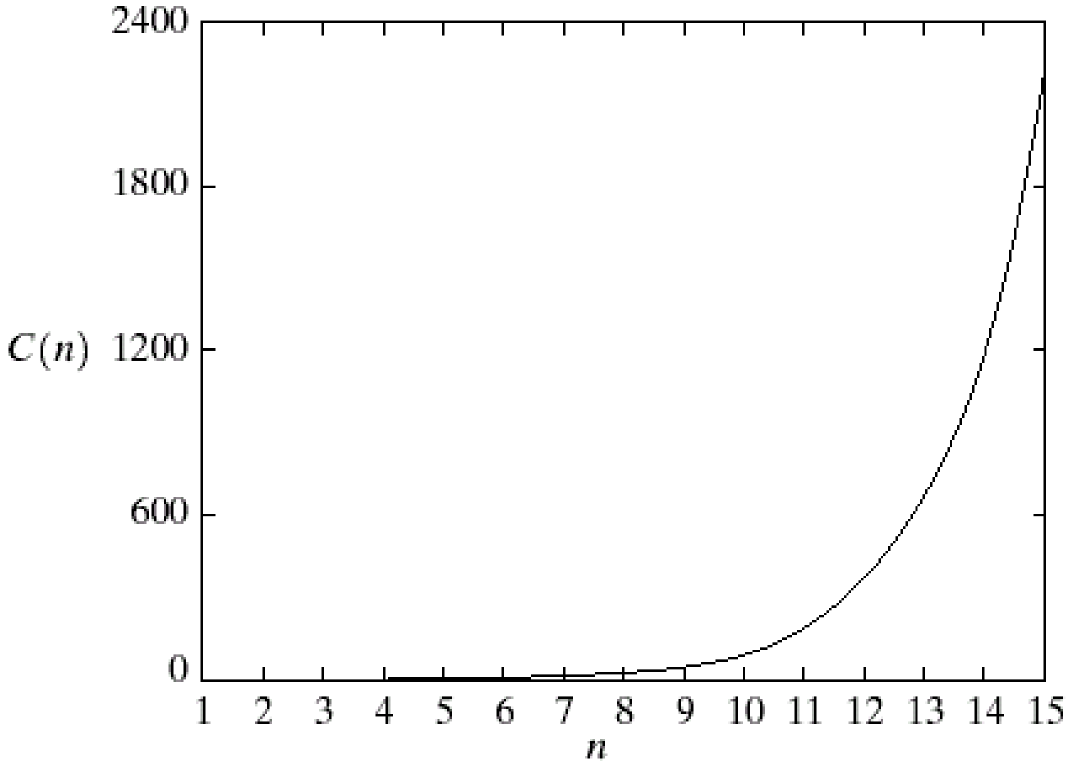


FIGURE 4.42
Computational advantage of the FFT over a direct implementation of the 1-D DFT. Note that the advantage increases rapidly as a function of *n*.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Relation Between Spatial and Frequency Resolutions

$$\Delta u = \frac{1}{M\Delta x}$$

$$\Delta v = \frac{1}{N\Delta y}$$

where

Δx = spatial resolution in x direction

Δy = spatial resolution in y direction

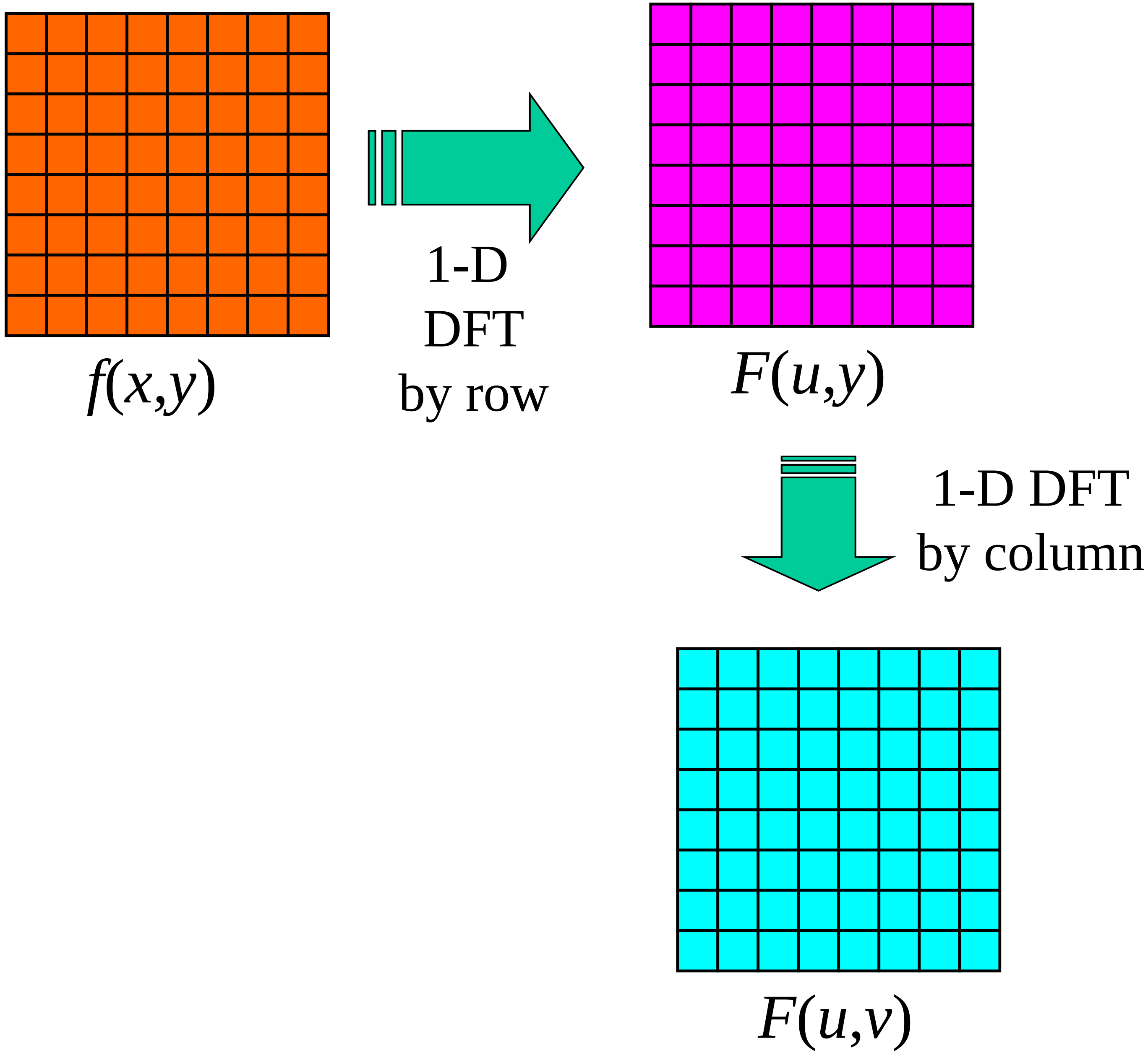
(Δx and Δy are pixel width and height.)

Δu = frequency resolution in x direction

Δv = frequency resolution in y direction

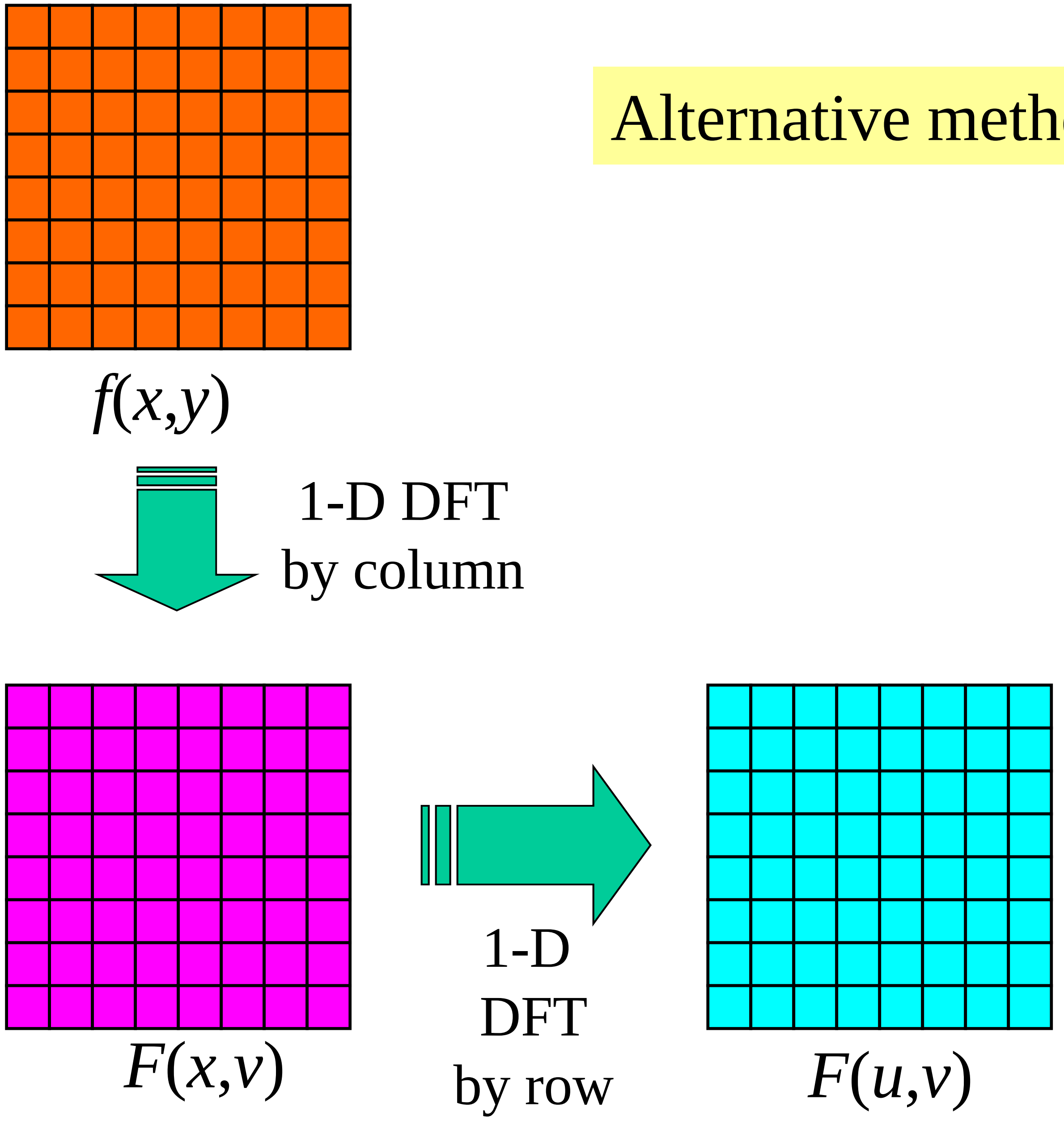
N,M = image width and height

How to Perform 2-D DFT by Using 1-D DFT



How to Perform 2-D DFT by Using 1-D DFT (cont.)

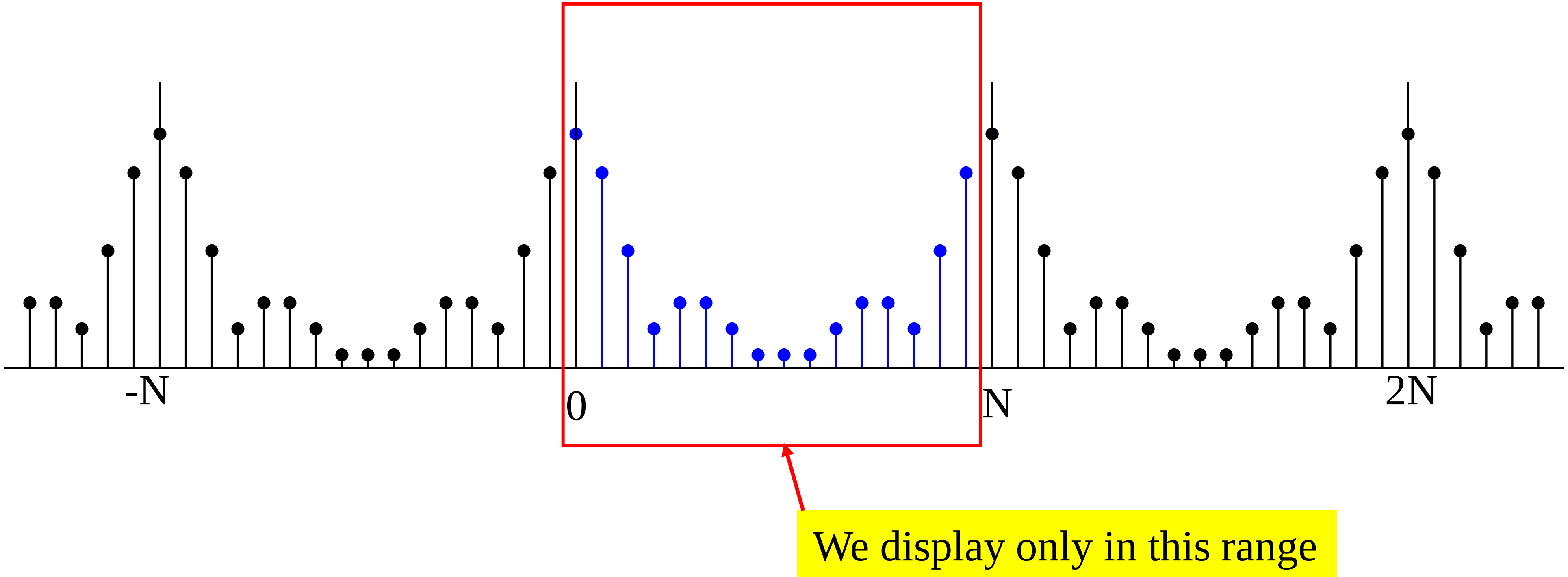
Alternative method



Periodicity of 1-D DFT

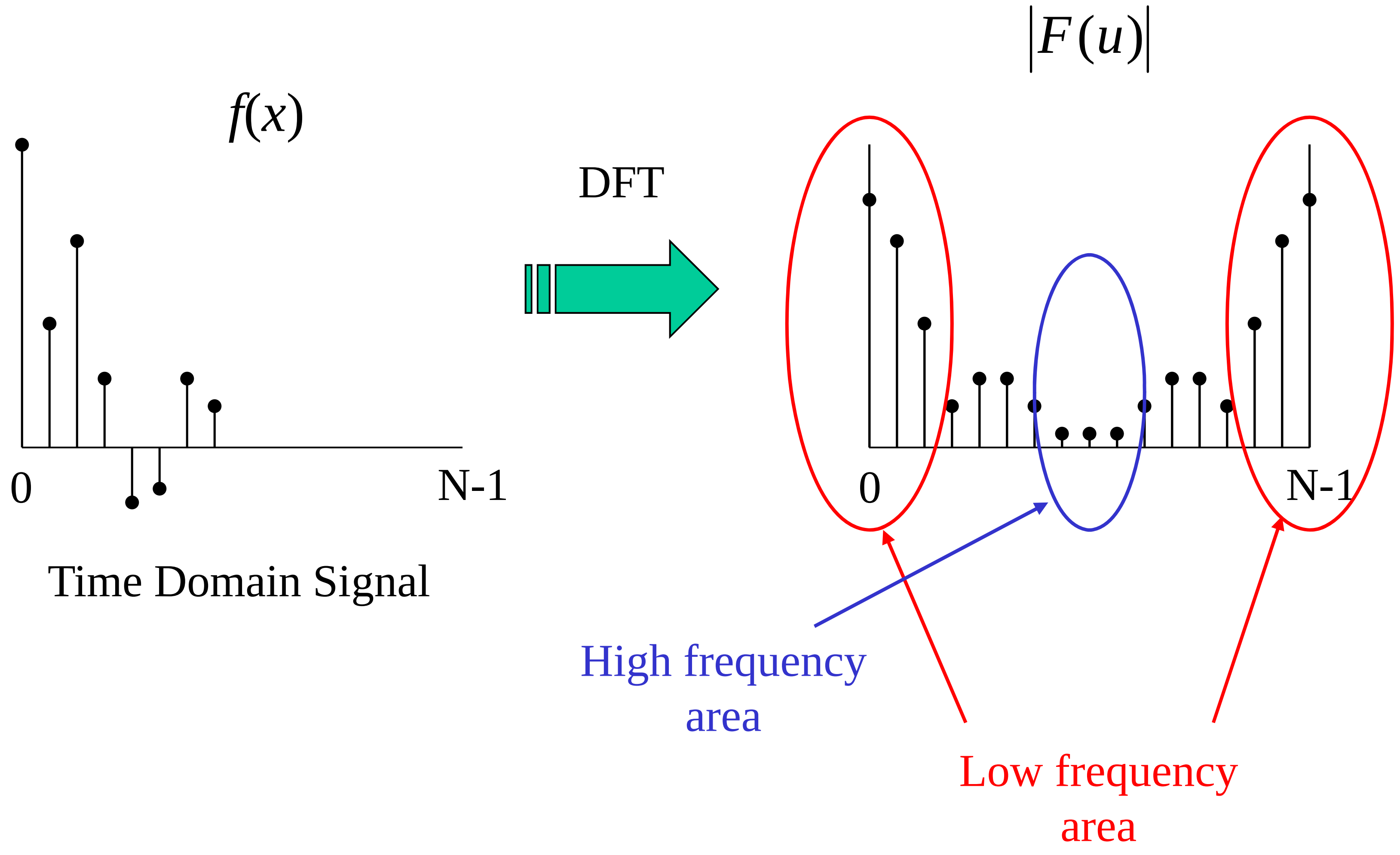
From DFT:

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/M}$$



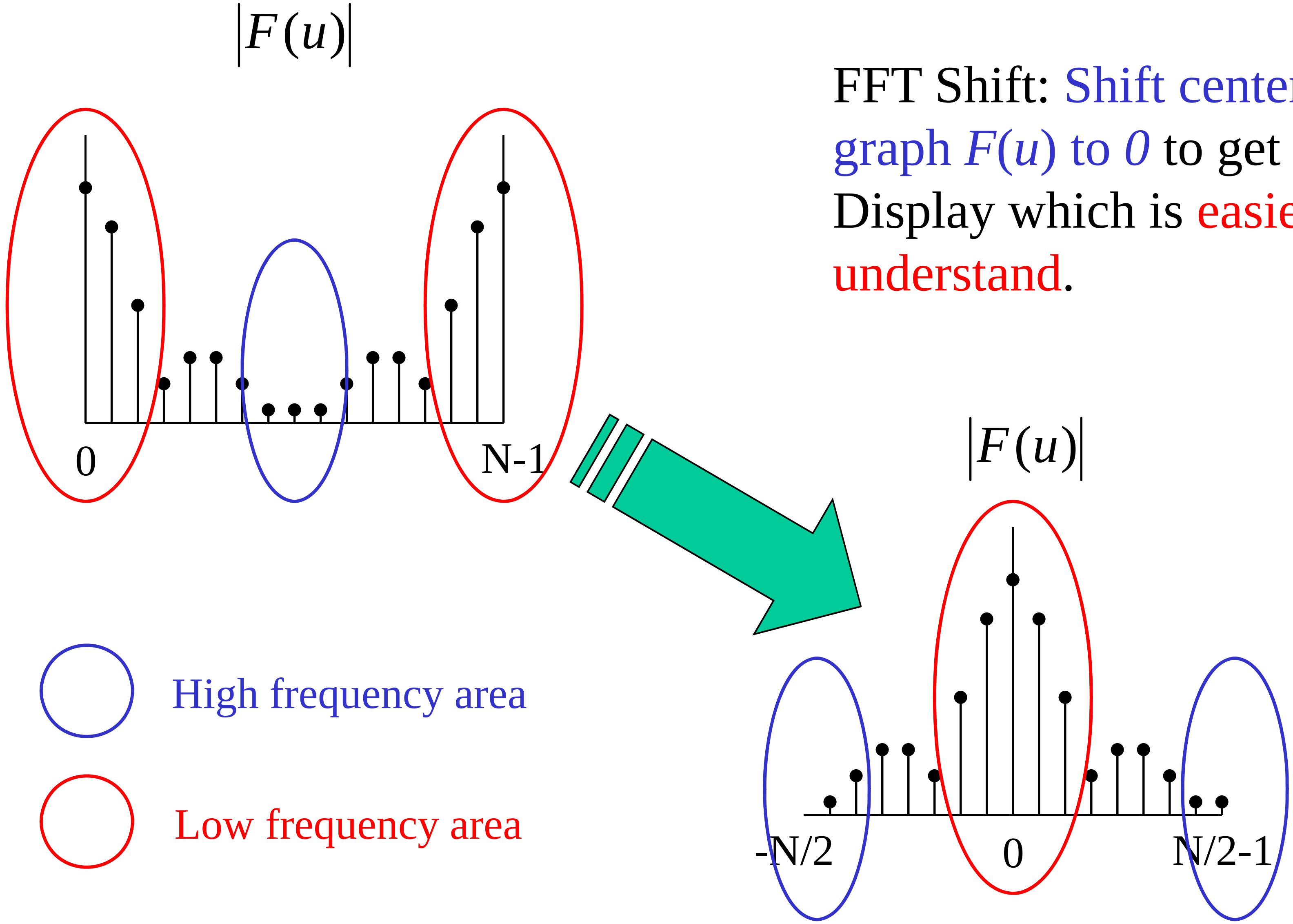
DFT repeats itself every N points (Period = N) but we usually display it for $n = 0 ,\ldots, N$-1

Conventional Display for 1-D DFT



The graph $F(u)$ is not easy to understand !

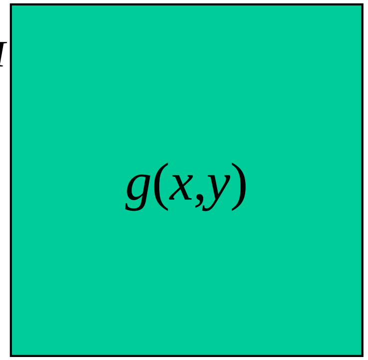
Conventional Display for DFT : FFT Shift

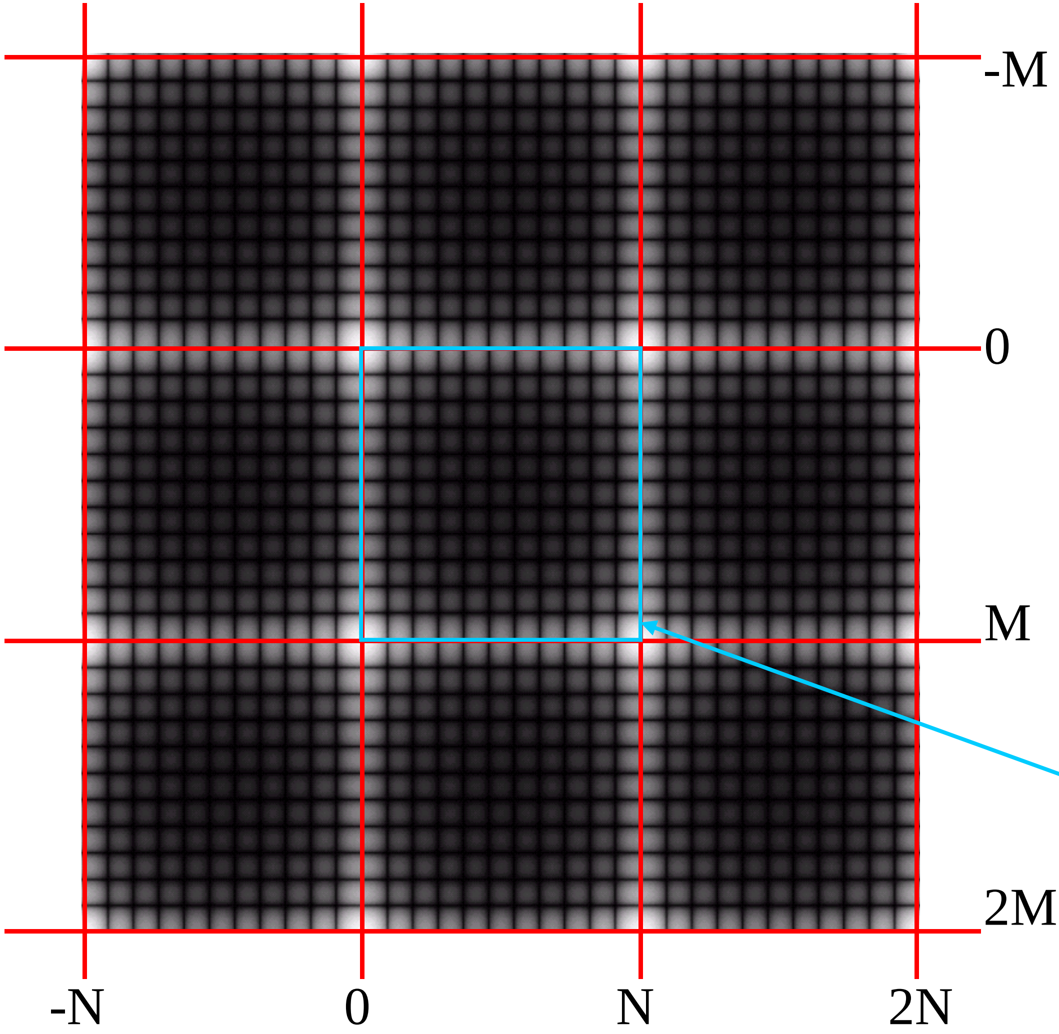


FFT Shift: Shift center of the graph $F(u)$ to 0 to get better Display which is easier to understand.

Periodicity of 2-D DFT

2-D DFT: $F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M}$





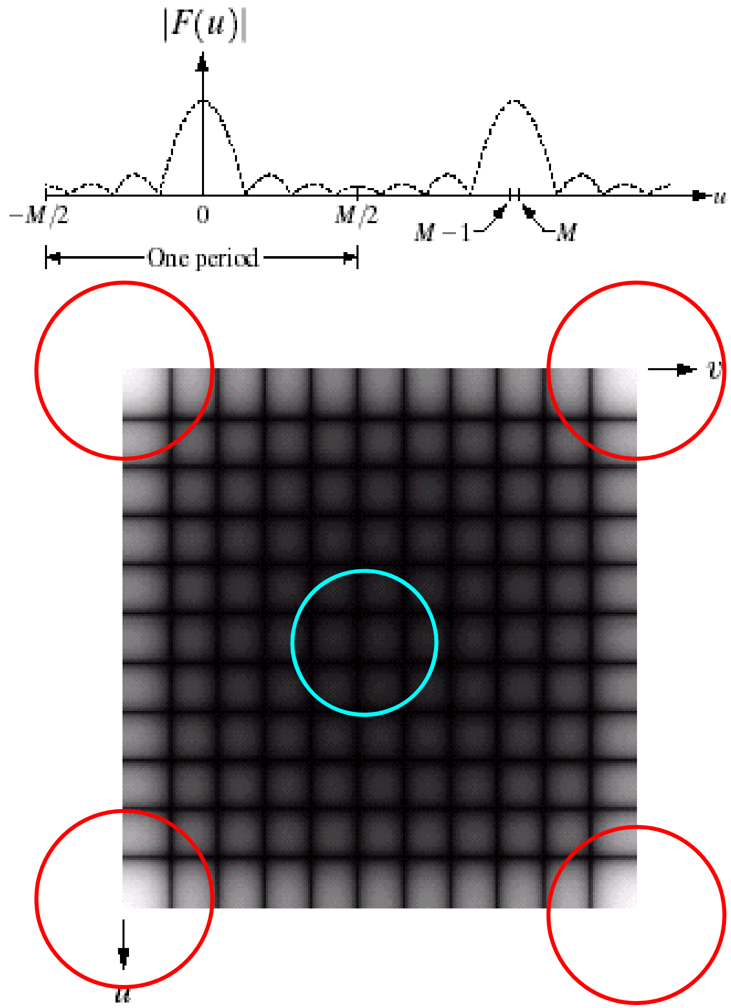
For an image of size *N*x*M* pixels, its 2-D DFT repeats itself every *N* points in *x*-direction and every *M* points in *y*-direction.

We display only in this range

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Conventional Display for 2-D DFT

$F(u,v)$ has low frequency areas at corners of the image while high frequency areas are at the center of the image which is inconvenient to interpret.



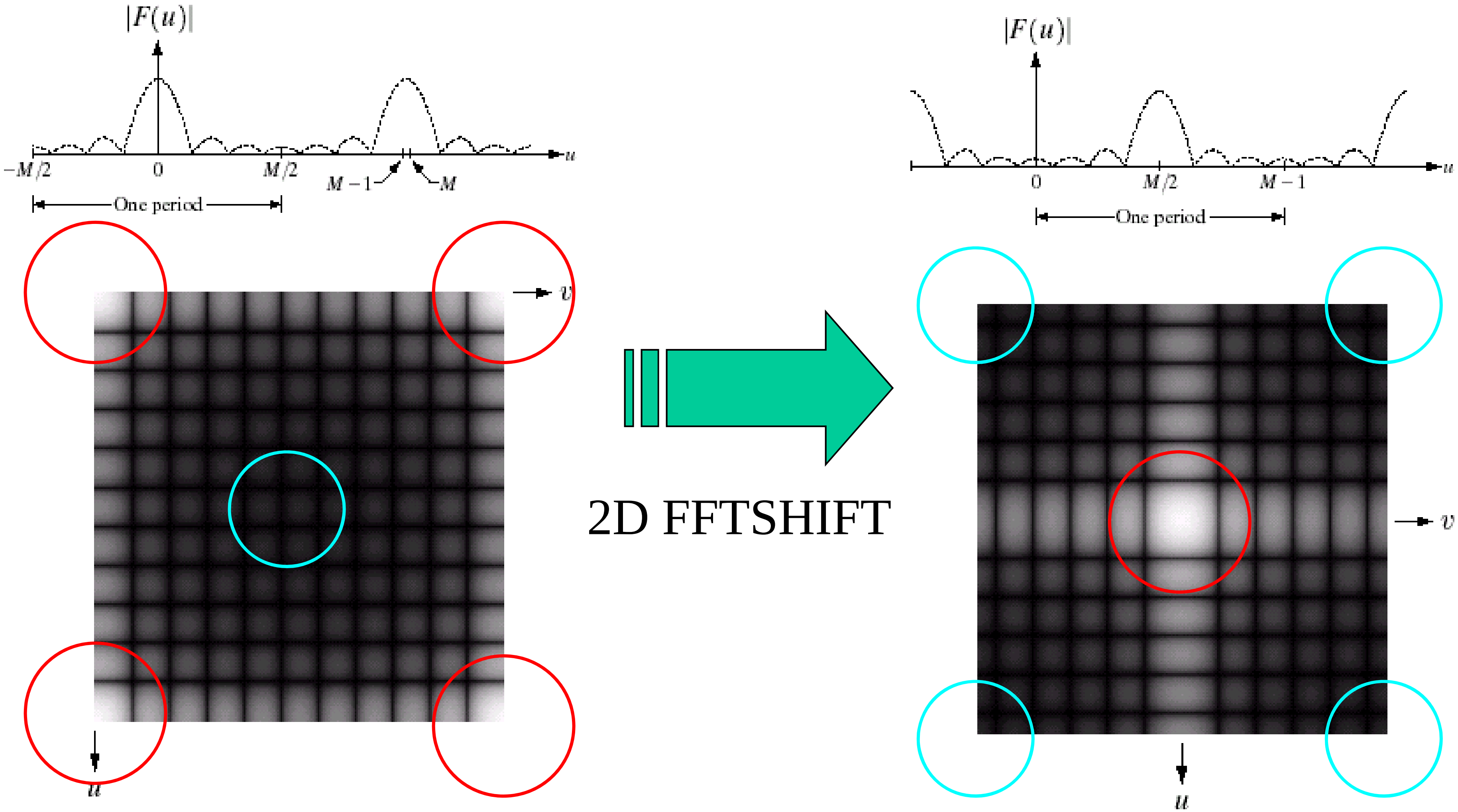
High frequency area

Low frequency area

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

2-D FFT Shift : Better Display of 2-D DFT

2-D FFT Shift is a MATLAB function: Shift the zero frequency of $F(u,v)$ to the center of an image.

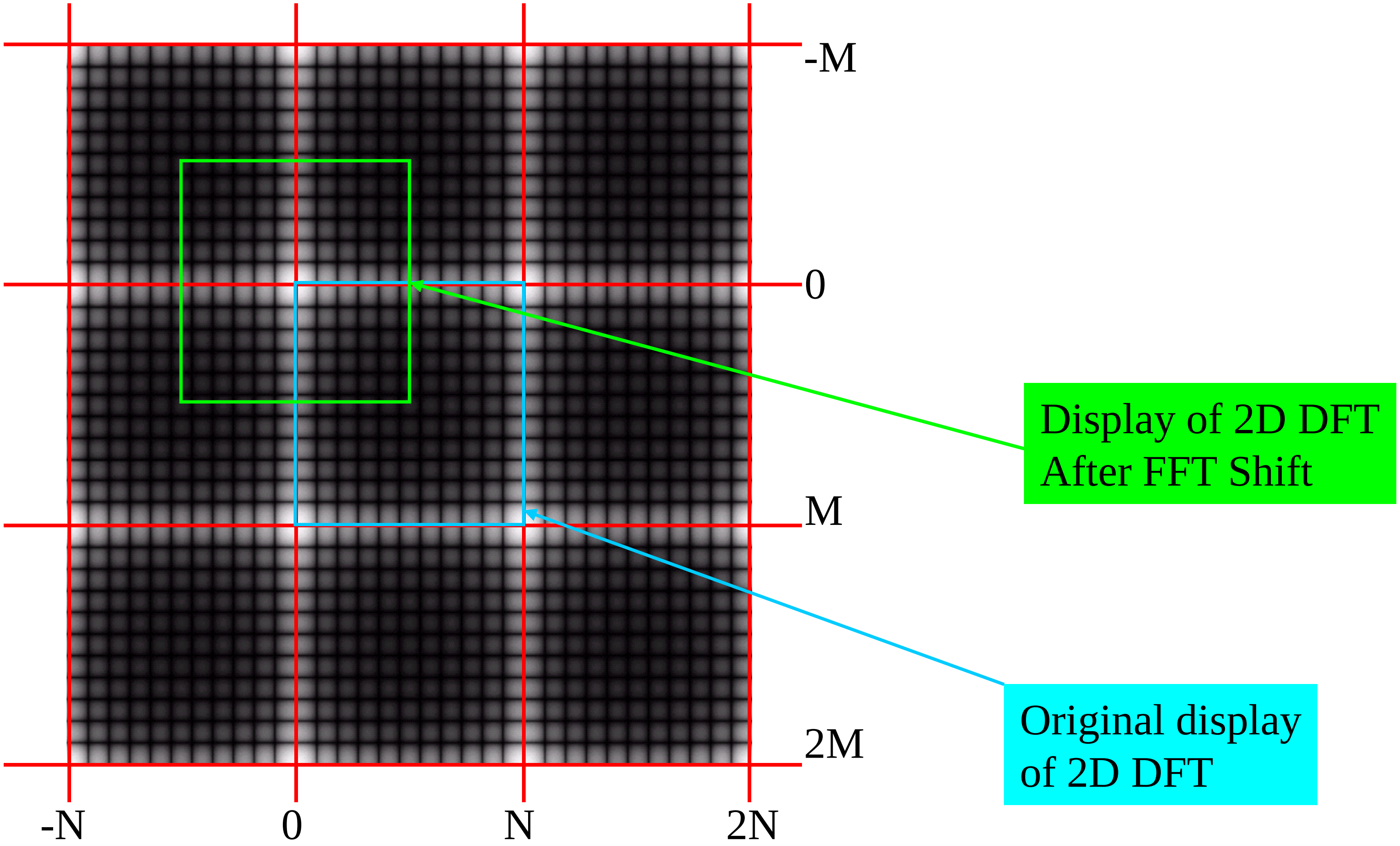


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

High frequency area

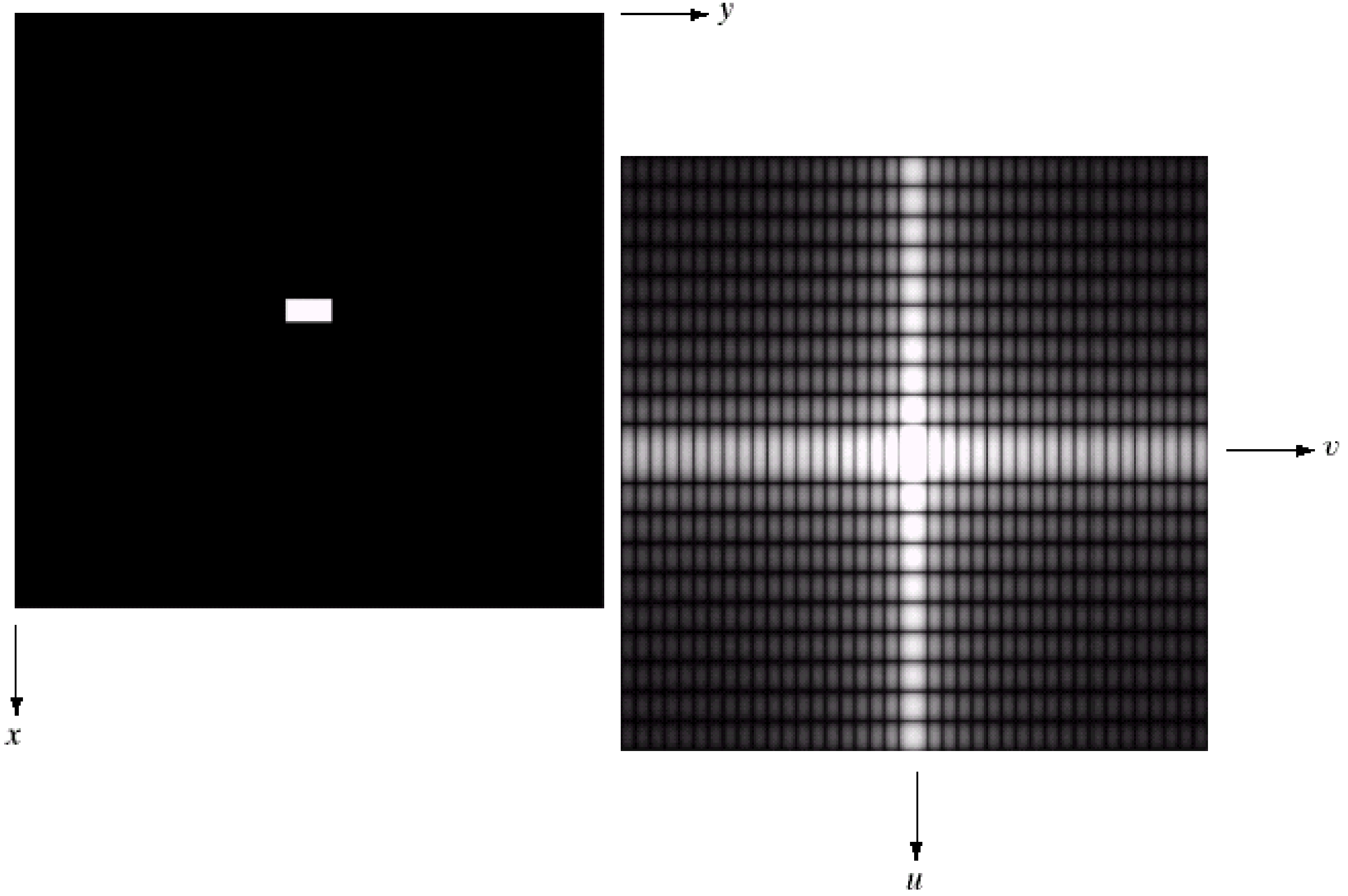
Low frequency area

2-D FFT Shift (cont.) : How it works



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of 2-D DFT



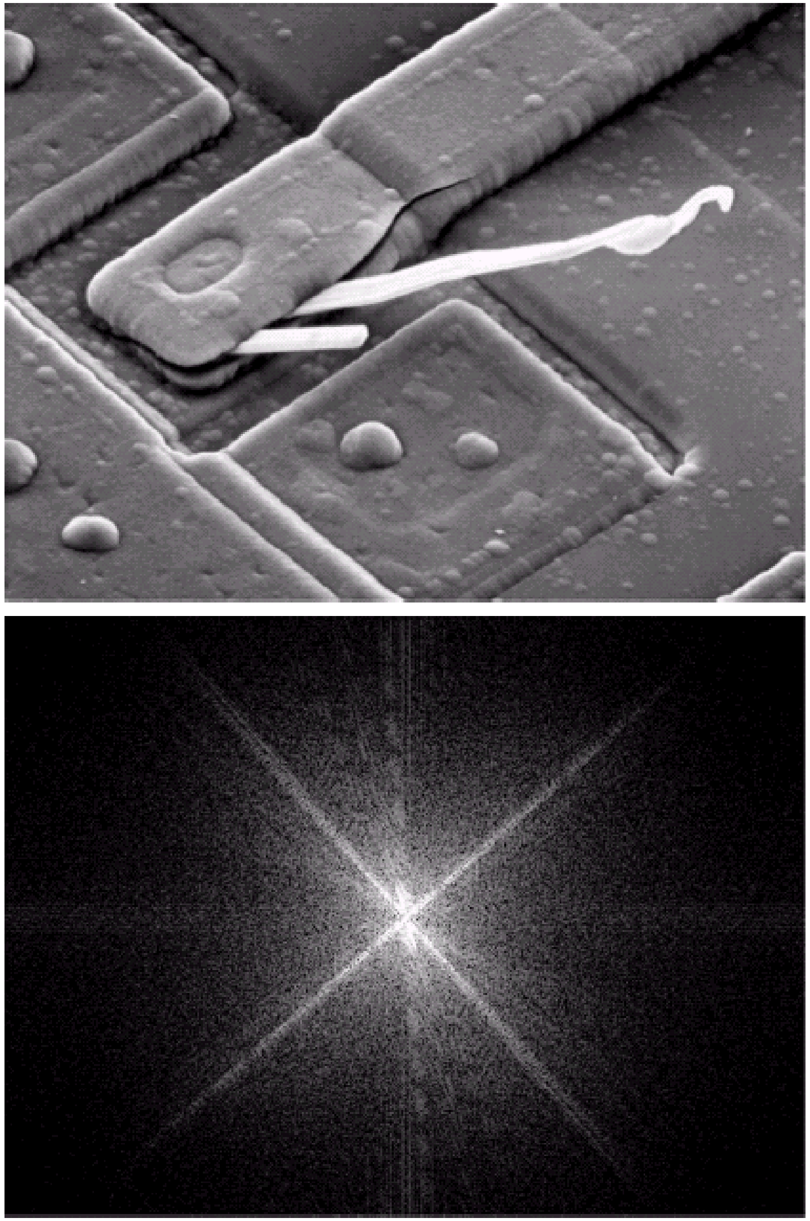
a b

FIGURE 4.3
(a) Image of a 20 × 40 white rectangle on a black background of size 512 × 512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.

Notice that the longer the time domain signal,
The shorter its Fourier transform

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of 2-D DFT



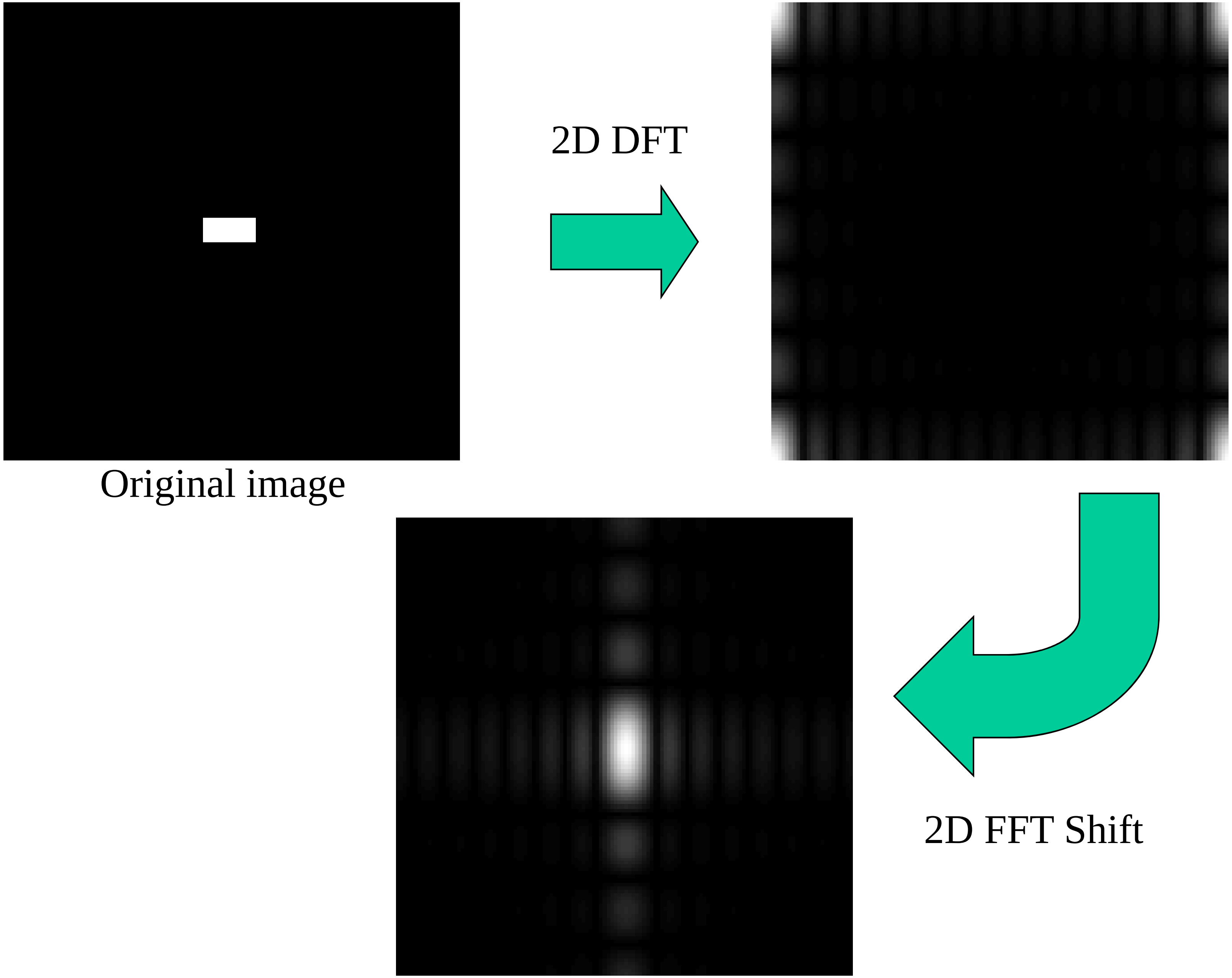
a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

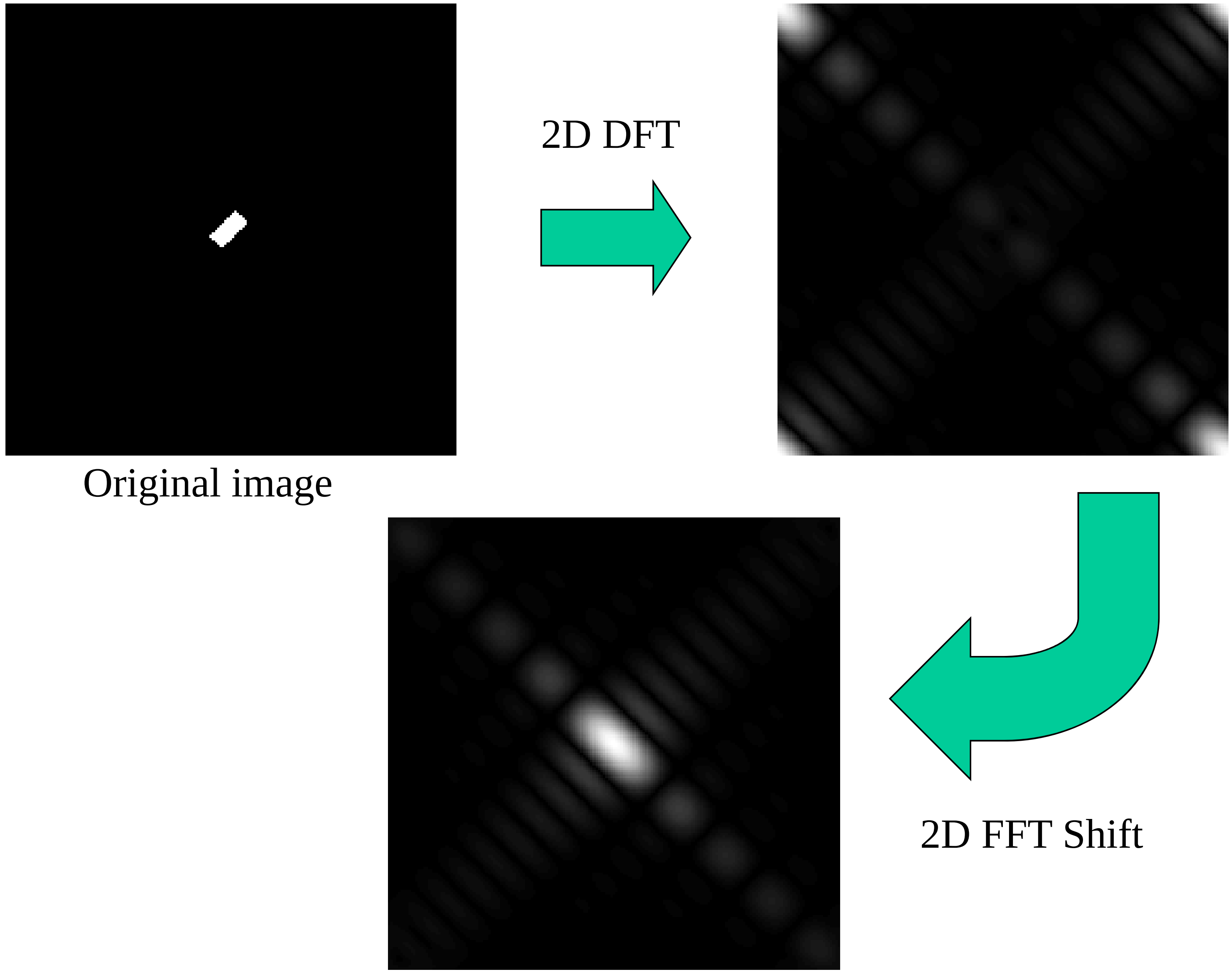
Notice that direction of an object in spatial image and Its Fourier transform are orthogonal to each other.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of 2-D DFT



Example of 2-D DFT

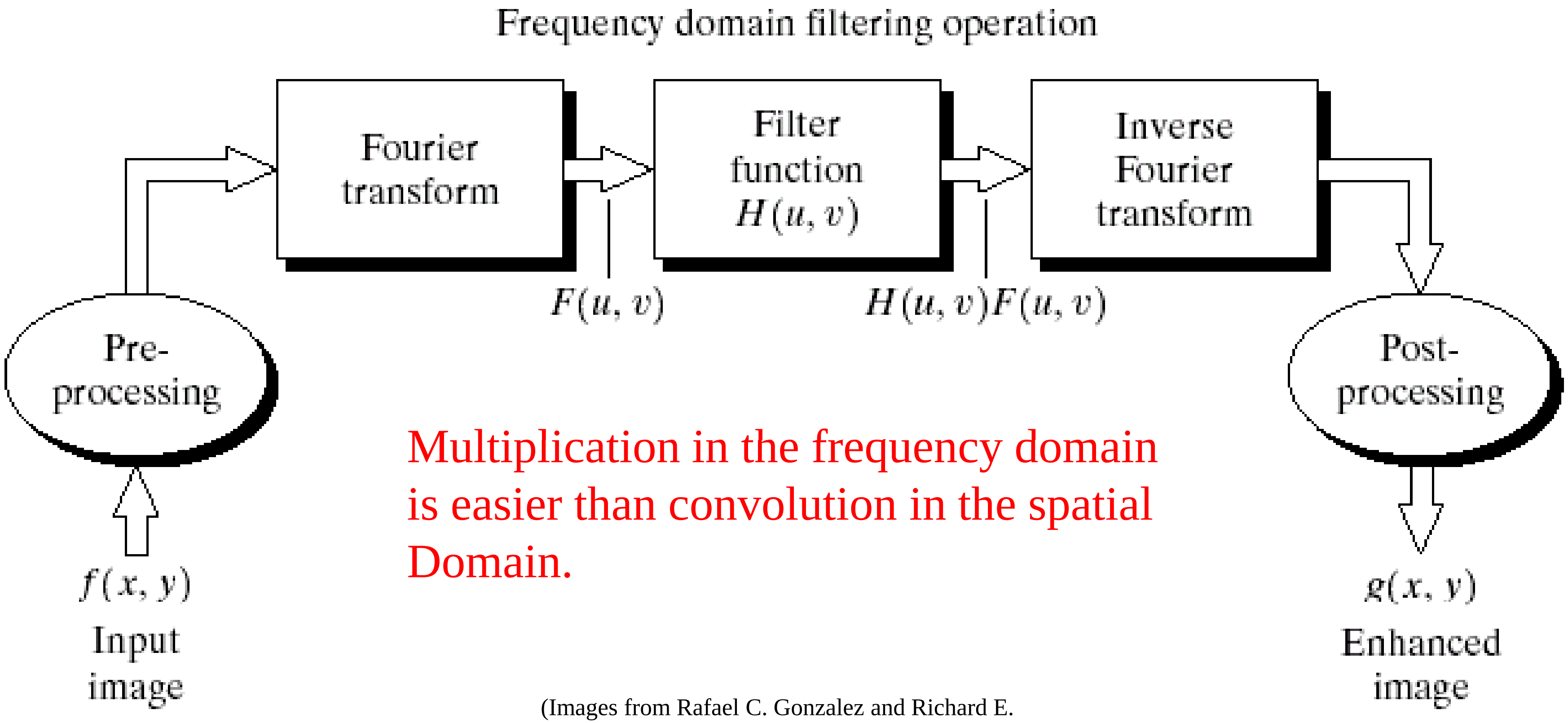


Basic Concept of Filtering in the Frequency Domain

From Fourier Transform Property:

$$g(x,y) = f(x,y) * h(x,y) \Leftrightarrow F(u,v) \cdot H(u,v) = G(u,v)$$

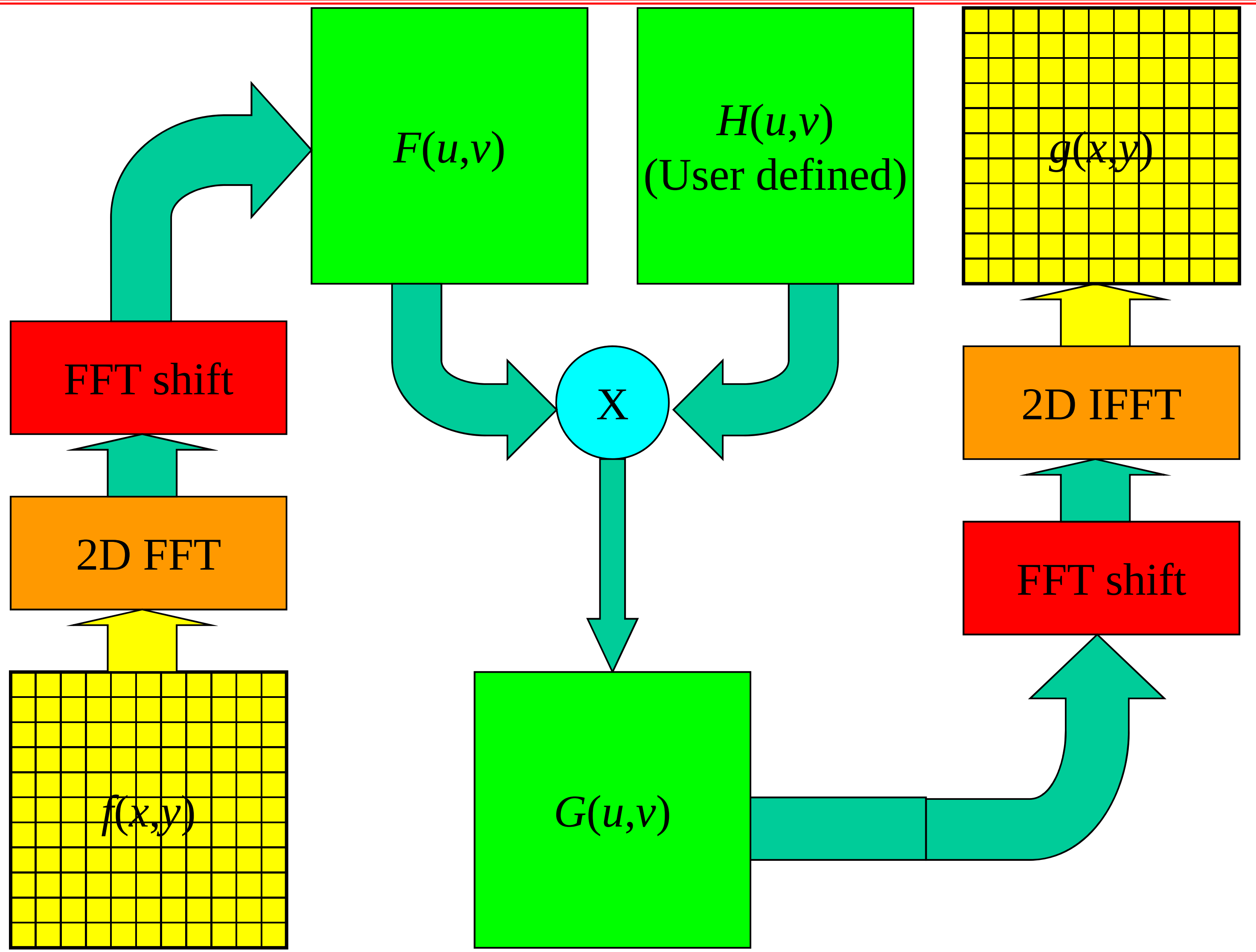
We cam perform filtering process by using



Multiplication in the frequency domain is easier than convolution in the spatial Domain.

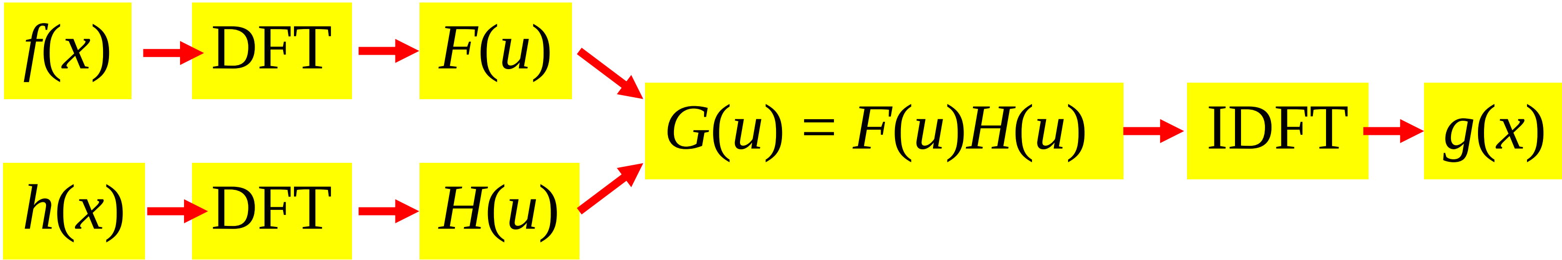
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Filtering in the Frequency Domain with FFT shift



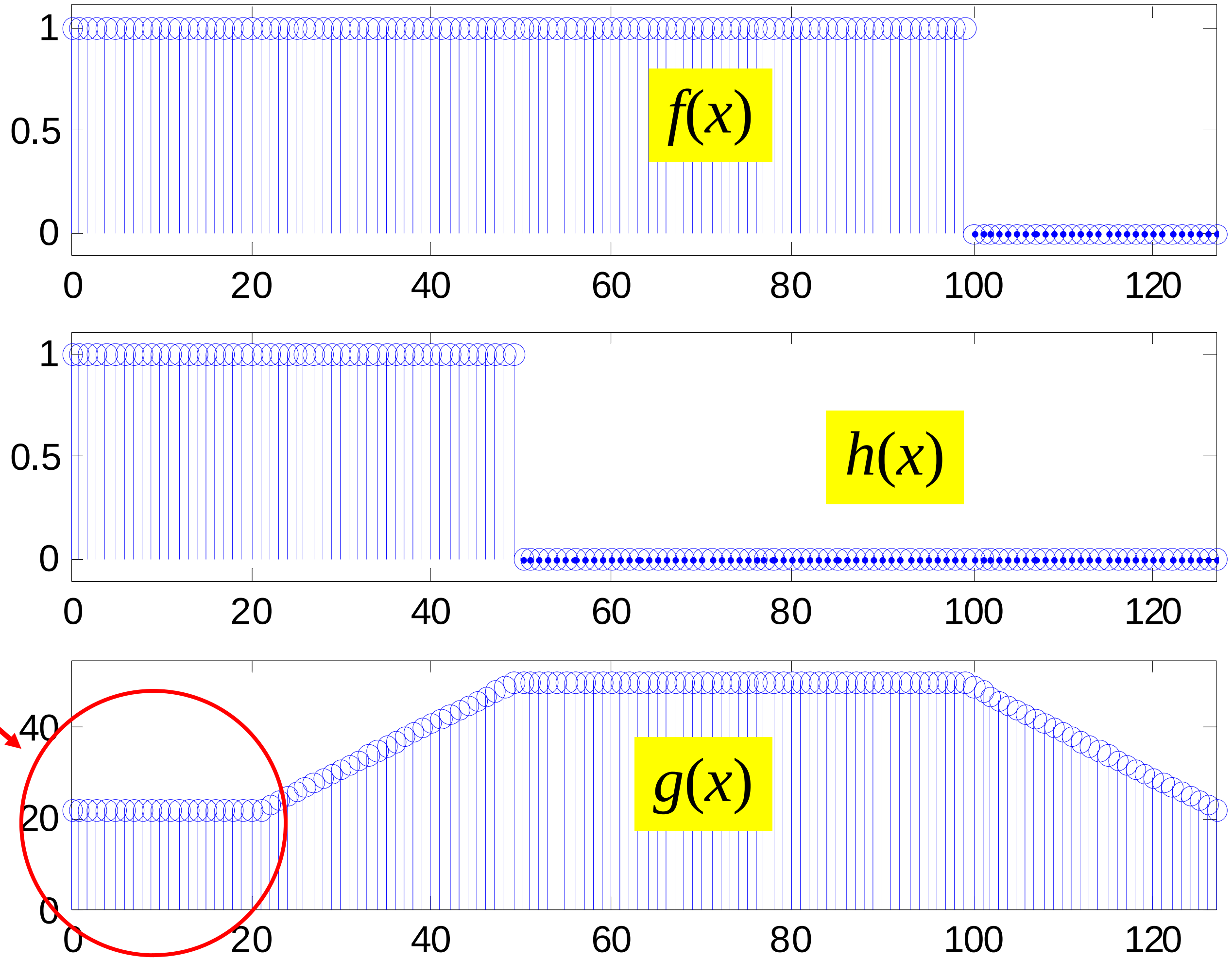
In this case, $F(u,v)$ and $H(u,v)$ must have the same size and have the zero frequency at the center.

Multiplication in Freq. Domain = Circular Convolution



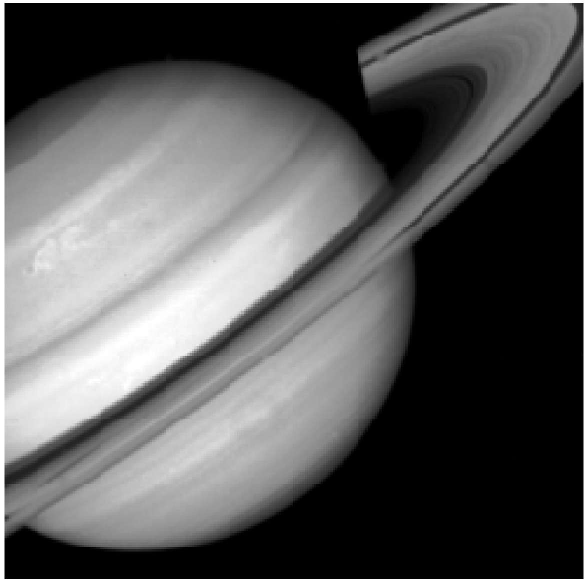
Multiplication of DFTs of 2 signals is equivalent to perform circular convolution in the spatial domain.

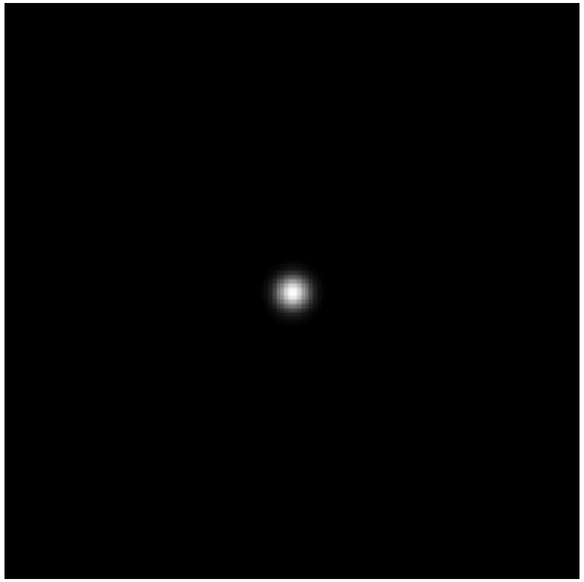
“Wrap around” effect



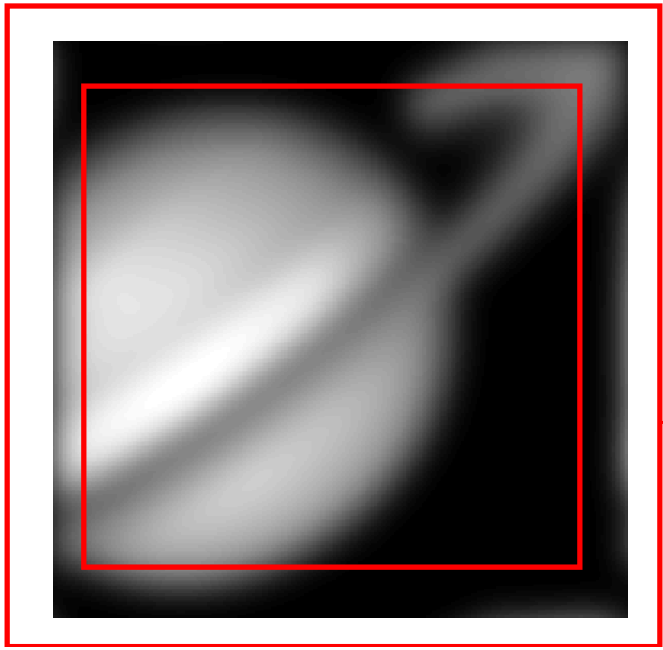
Multiplication in Freq. Domain = Circular Convolution

Original image





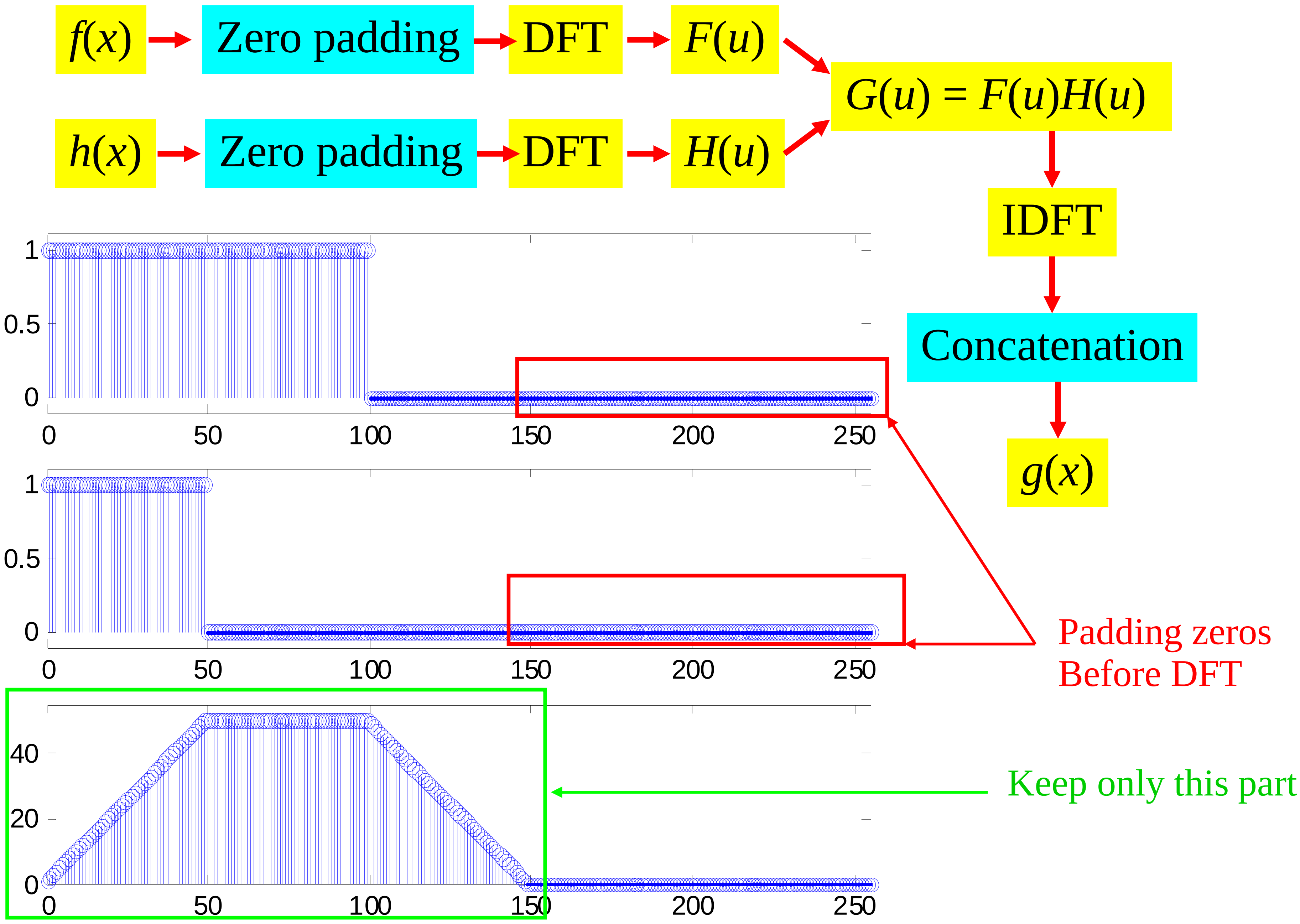
H(*u*,*v*)
Gaussian
Lowpass
Filter with
D0 = 5



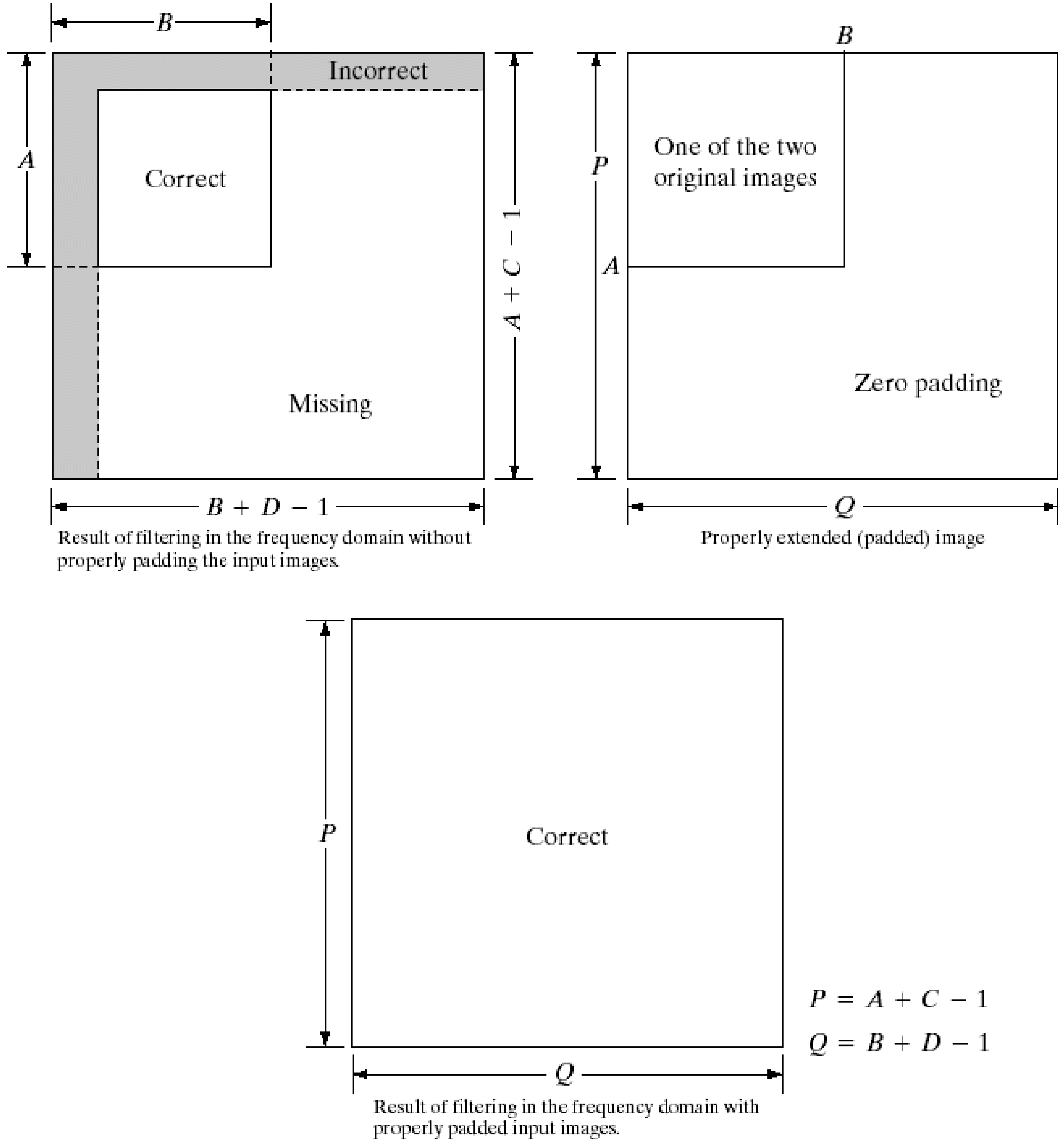
Filtered image
(obtained using
circular convolution)

Incorrect areas at image rims

Linear Convolution by using Circular Convolution and Zero Padding



Linear Convolution by using Circular Convolution and Zero Padding

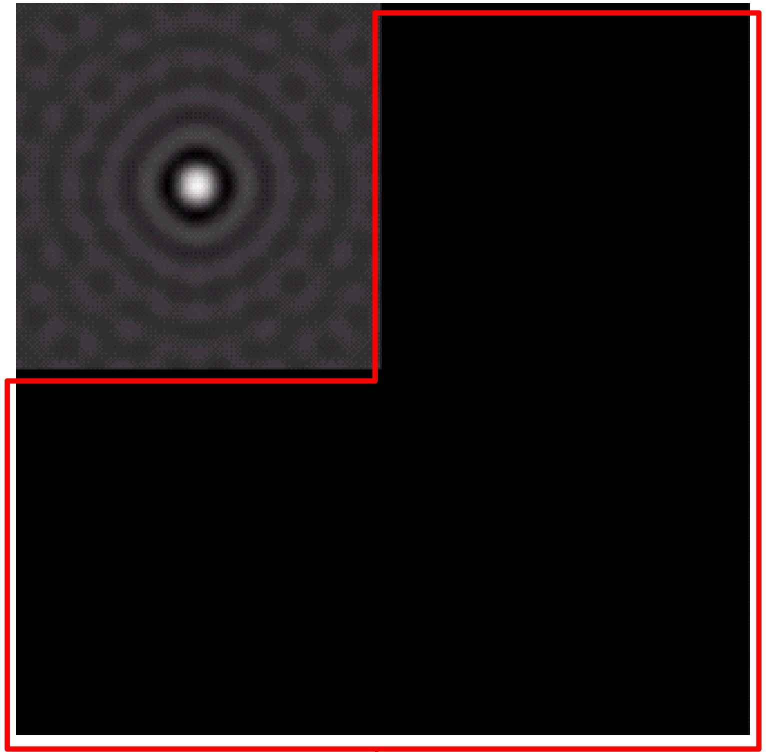


a b
c

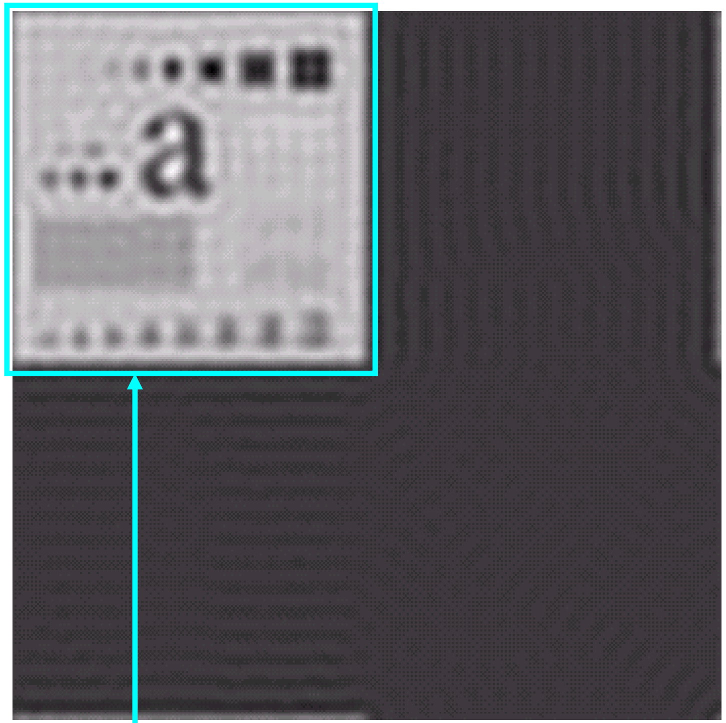
FIGURE 4.38 Illustration of the need for function padding. (a) Result of performing 2-D convolution without padding. (b) Proper function padding. (c) Correct convolution result.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Linear Convolution by using Circular Convolution and Zero Padding



Zero padding area in the spatial
Domain of the mask image
(the ideal lowpass filter)

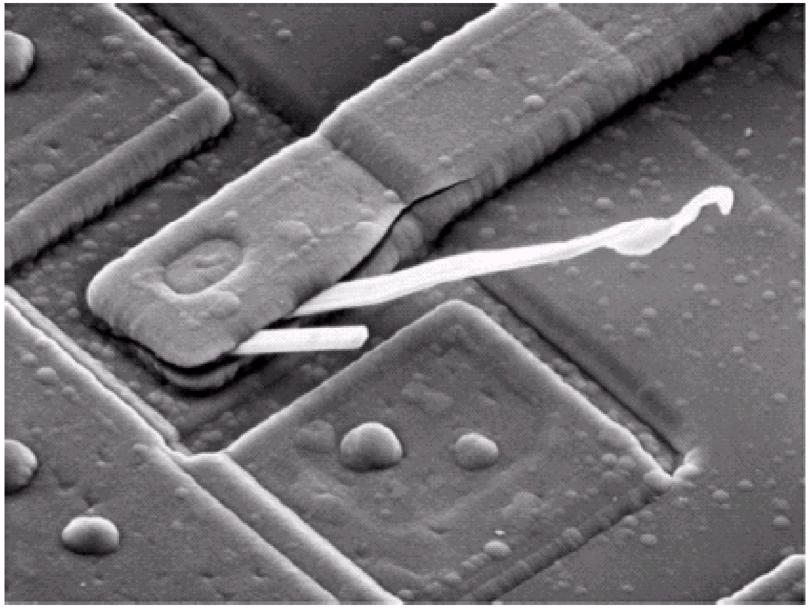


Filtered image

Only this area is kept.

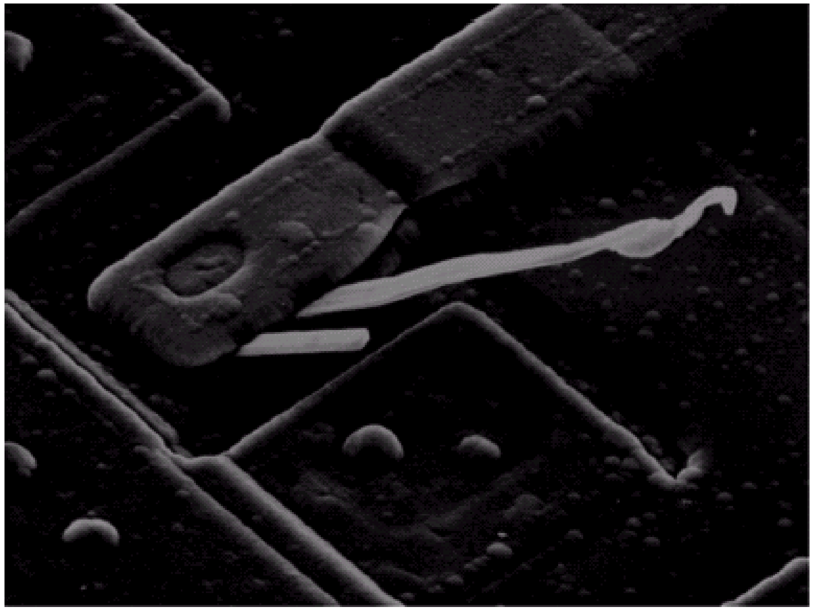
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Filtering in the Frequency Domain : Example



In this example, we set F(0,0) to zero which means that the zero frequency component is removed.

Note: Zero frequency = average intensity of an image

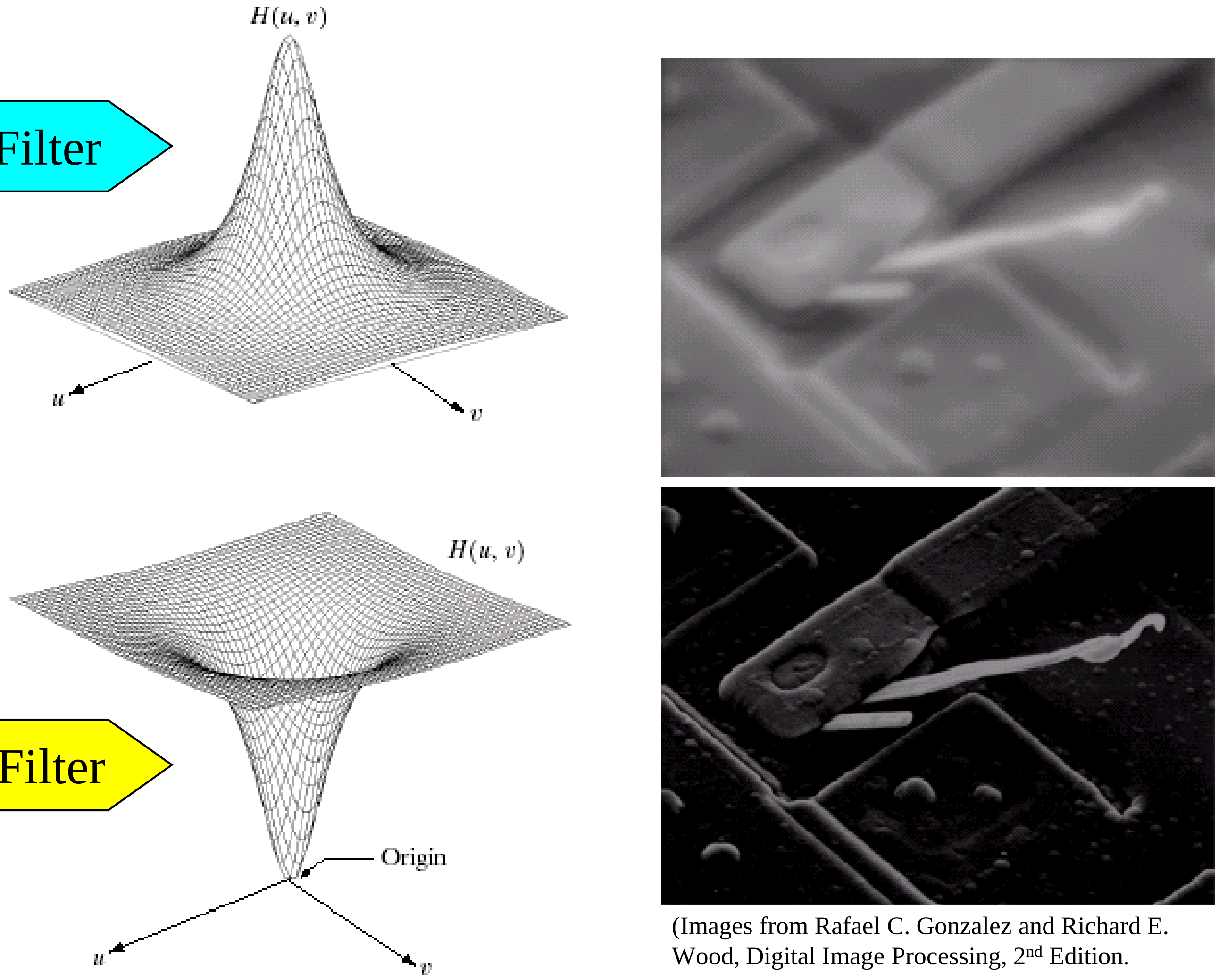


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Filtering in the Frequency Domain : Example

Lowpass Filter

Highpass Filter



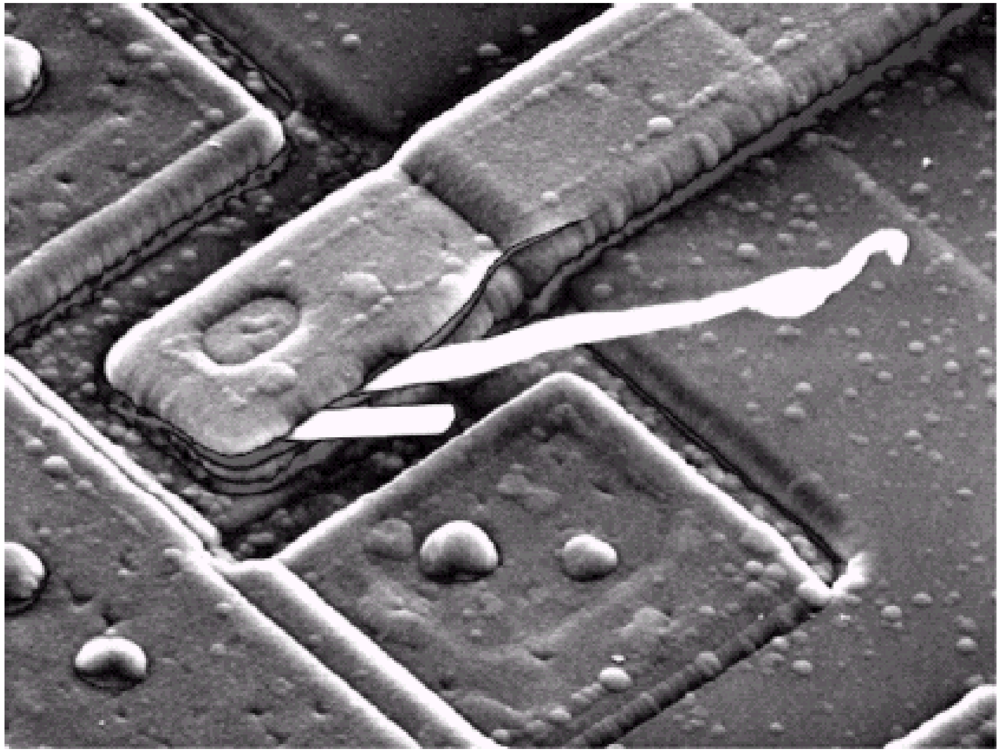
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Filtering in the Frequency Domain : Example (cont.)

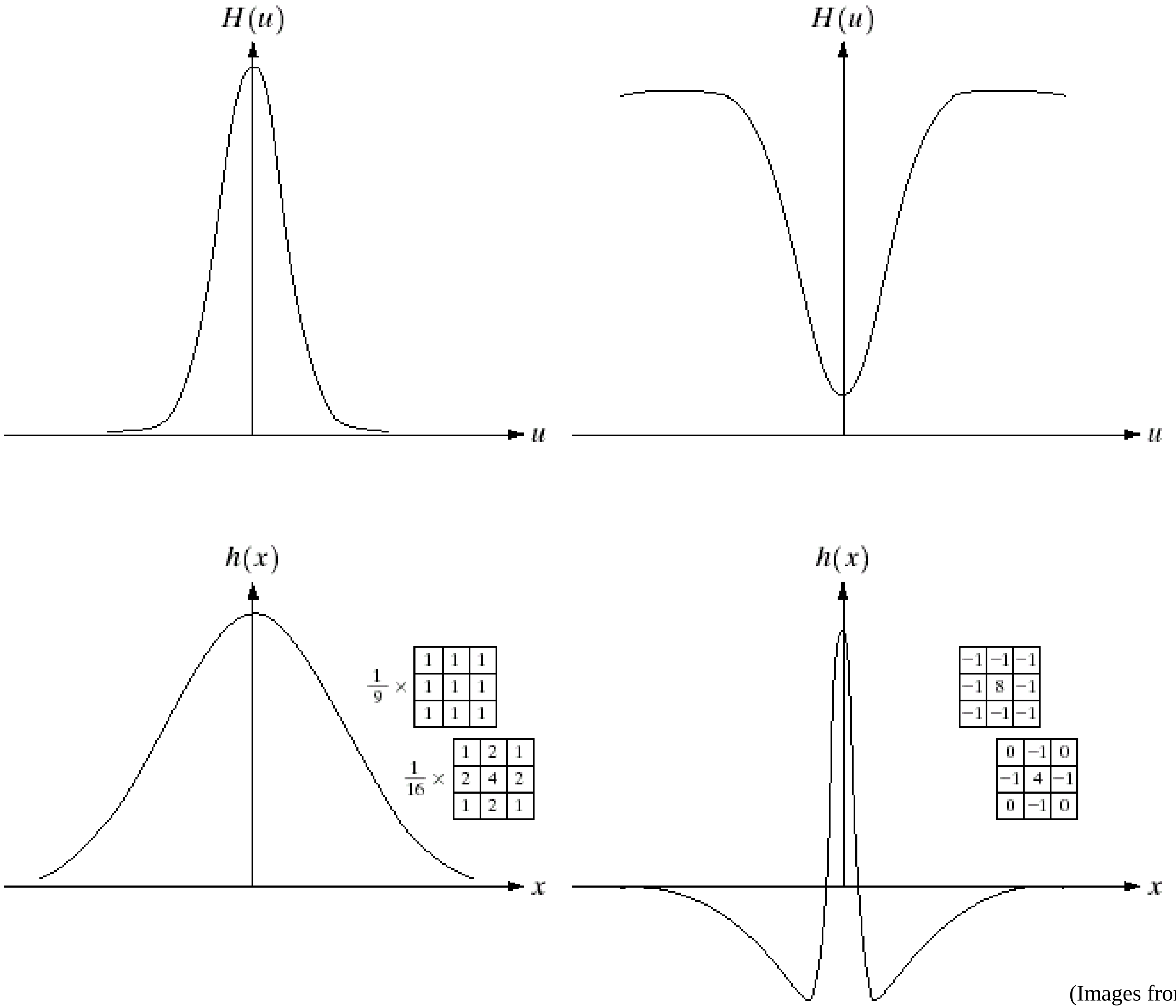
FIGURE 4.8
Result of highpass filtering the image in Fig. 4.4(a) with the filter in Fig. 4.7(c), modified by adding a constant of one-half the filter height to the filter function. Compare with Fig. 4.4(a).



Result of Sharpening Filter

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Filter Masks and Their Fourier Transforms



a b
c d

FIGURE 4.9
(a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

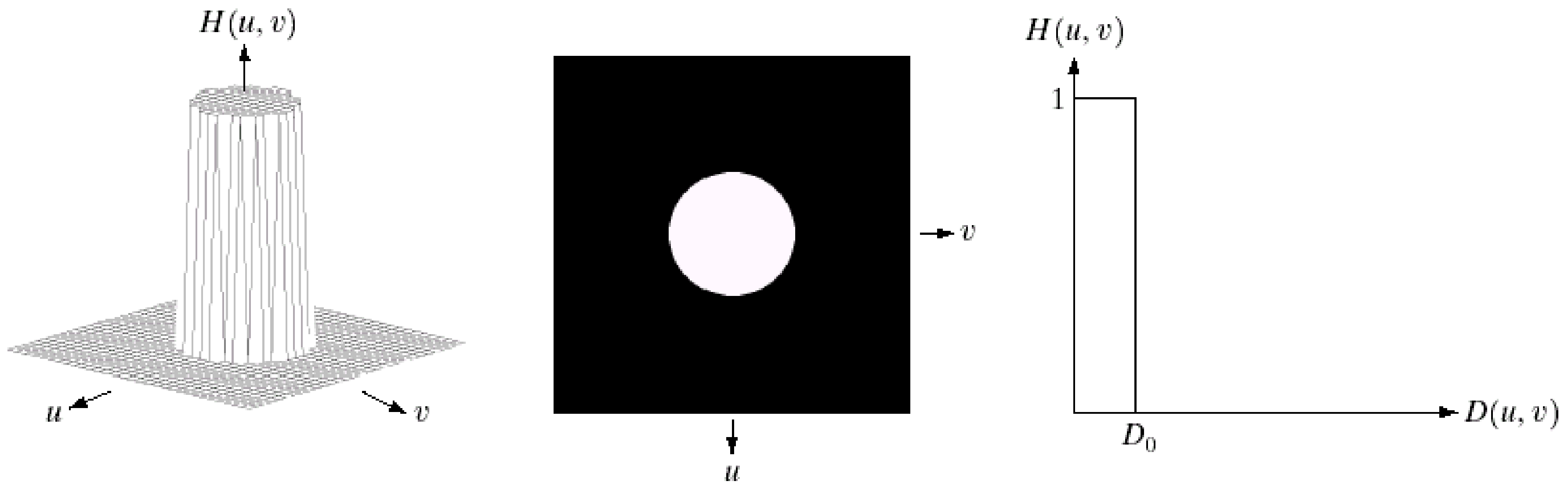
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Ideal Lowpass Filter

Ideal LPF Filter Transfer function

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ = Distance from (u,v) to the center of the mask.

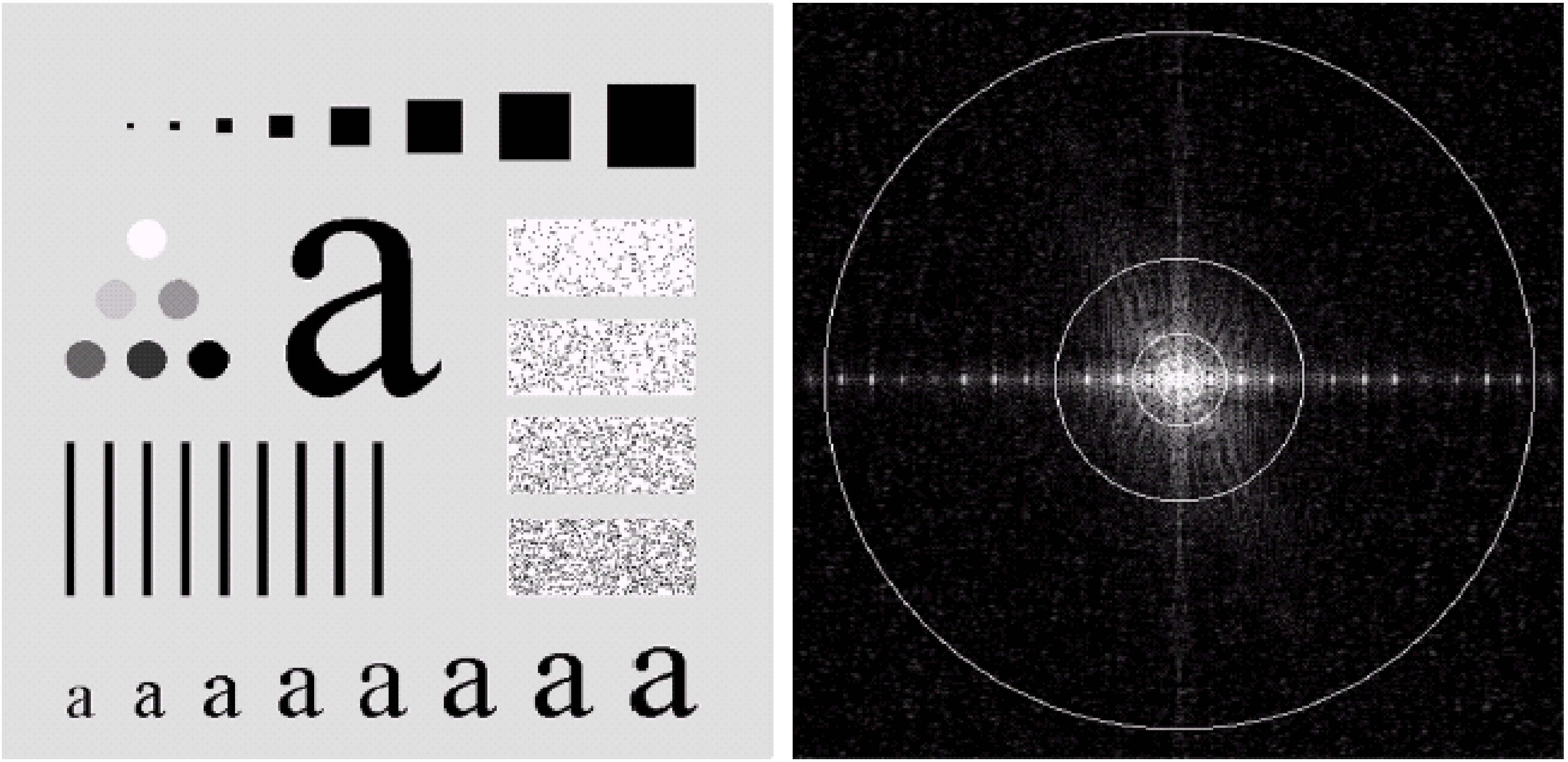


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Examples of Ideal Lowpass Filters



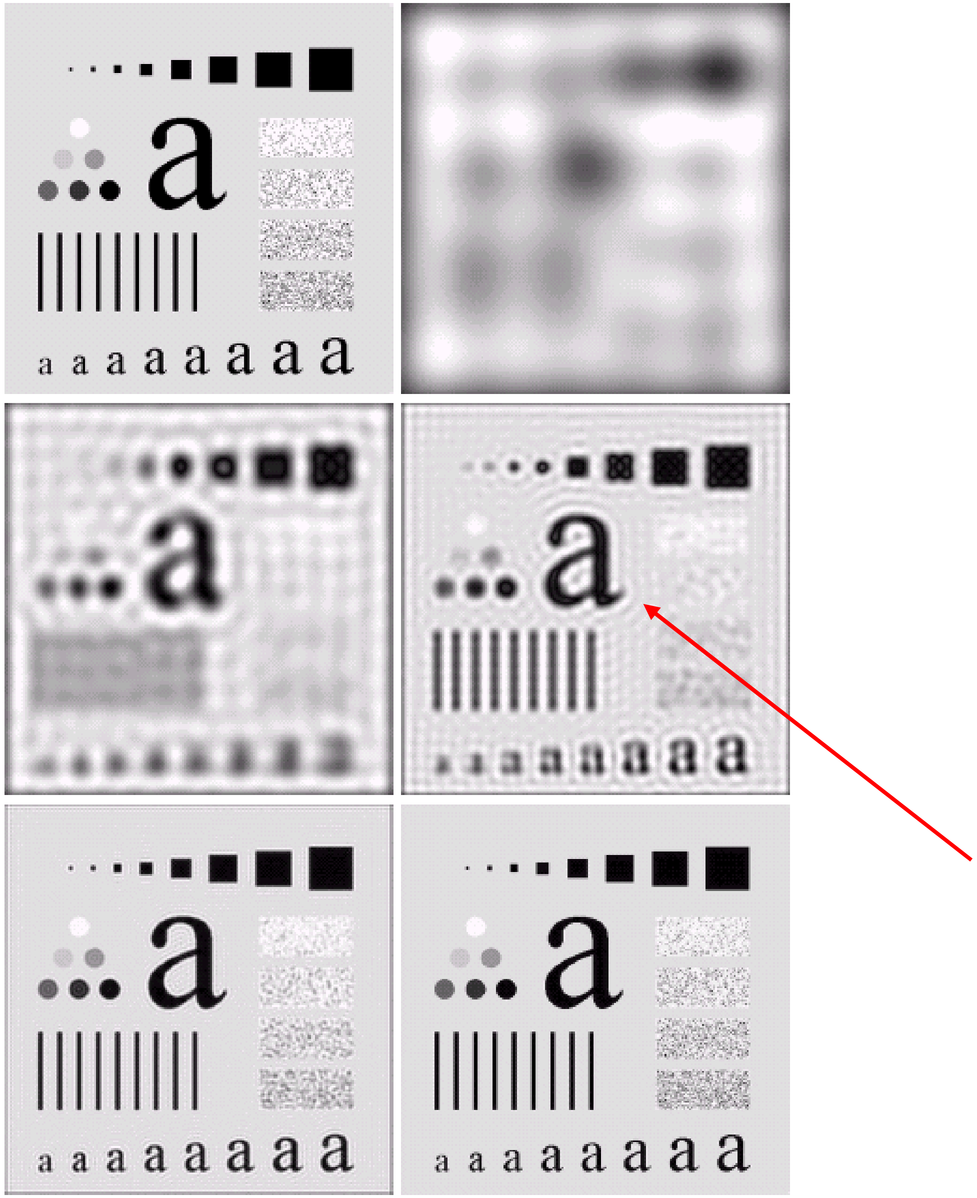
a b

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

The smaller D_0, the more high frequency components are removed.

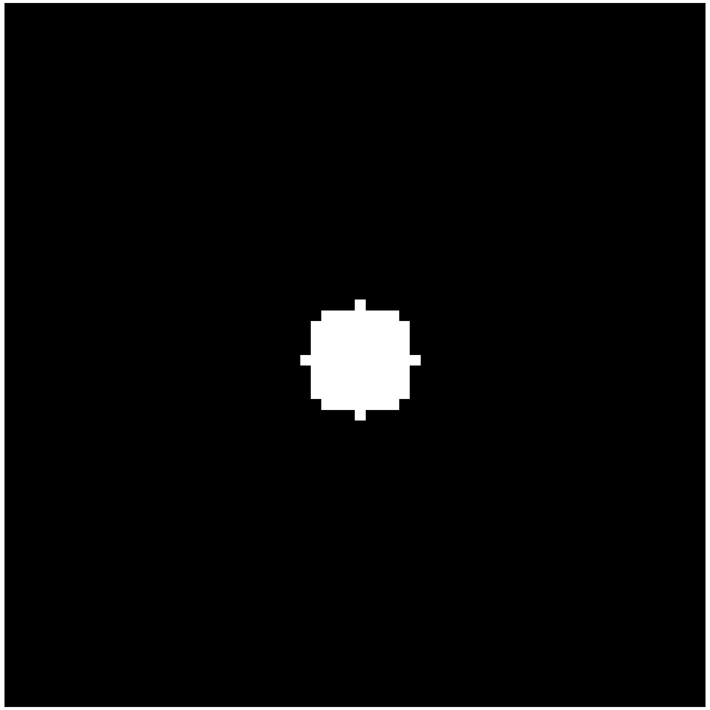
Results of Ideal Lowpass Filters



Ringing effect can be obviously seen!

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

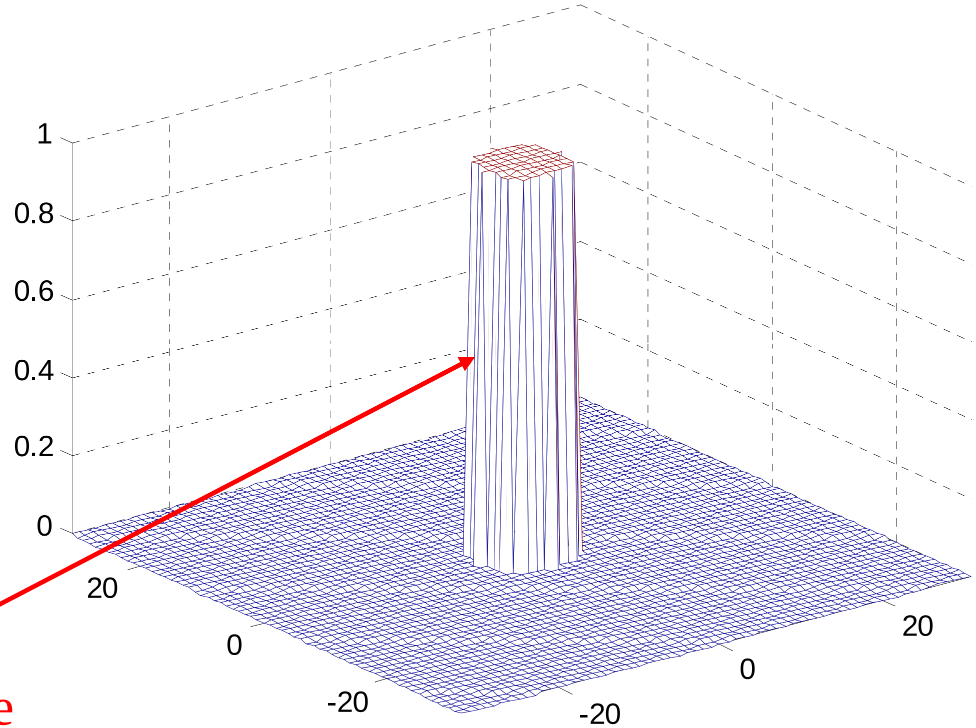
How ringing effect happens



Ideal Lowpass Filter
with $D_0 = 5$

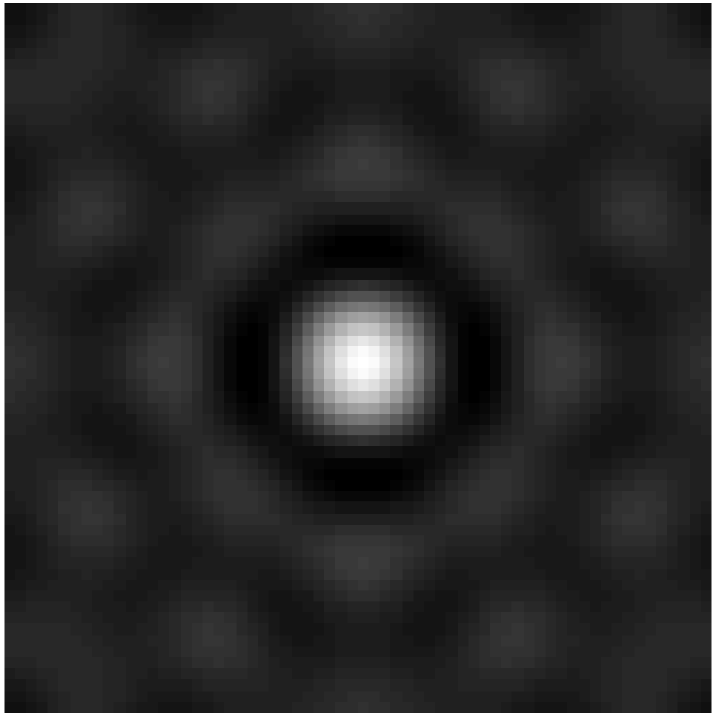
$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$$

Surface Plot



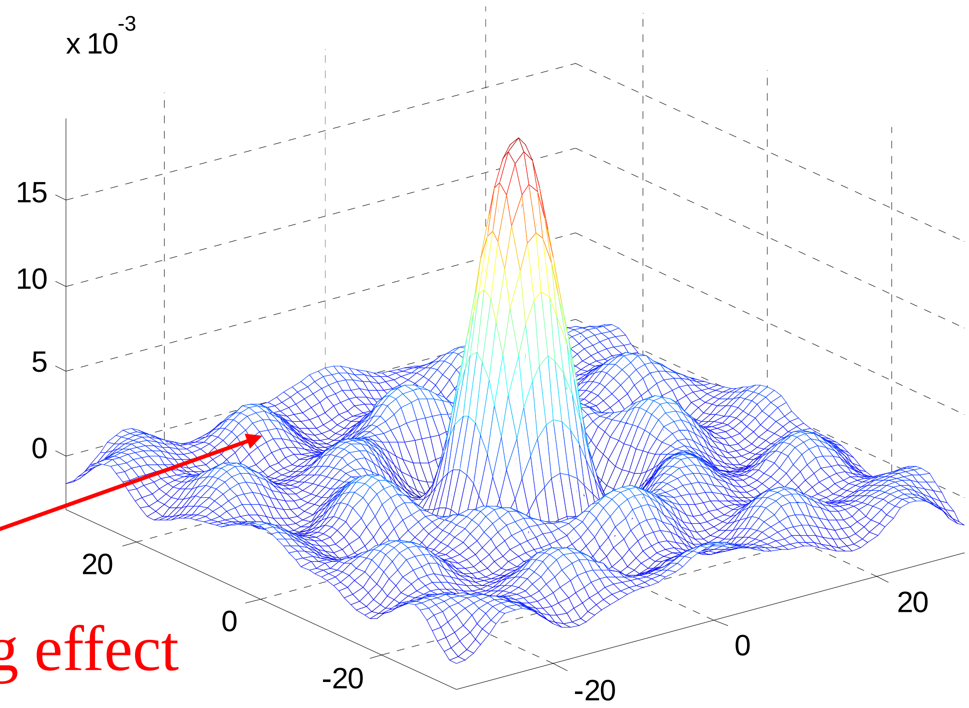
Abrupt change in the amplitude

How ringing effect happens (cont.)



Spatial Response of Ideal Lowpass Filter with $D_0 = 5$

Surface Plot



Ripples that cause ringing effect

How ringing effect happens (cont.)

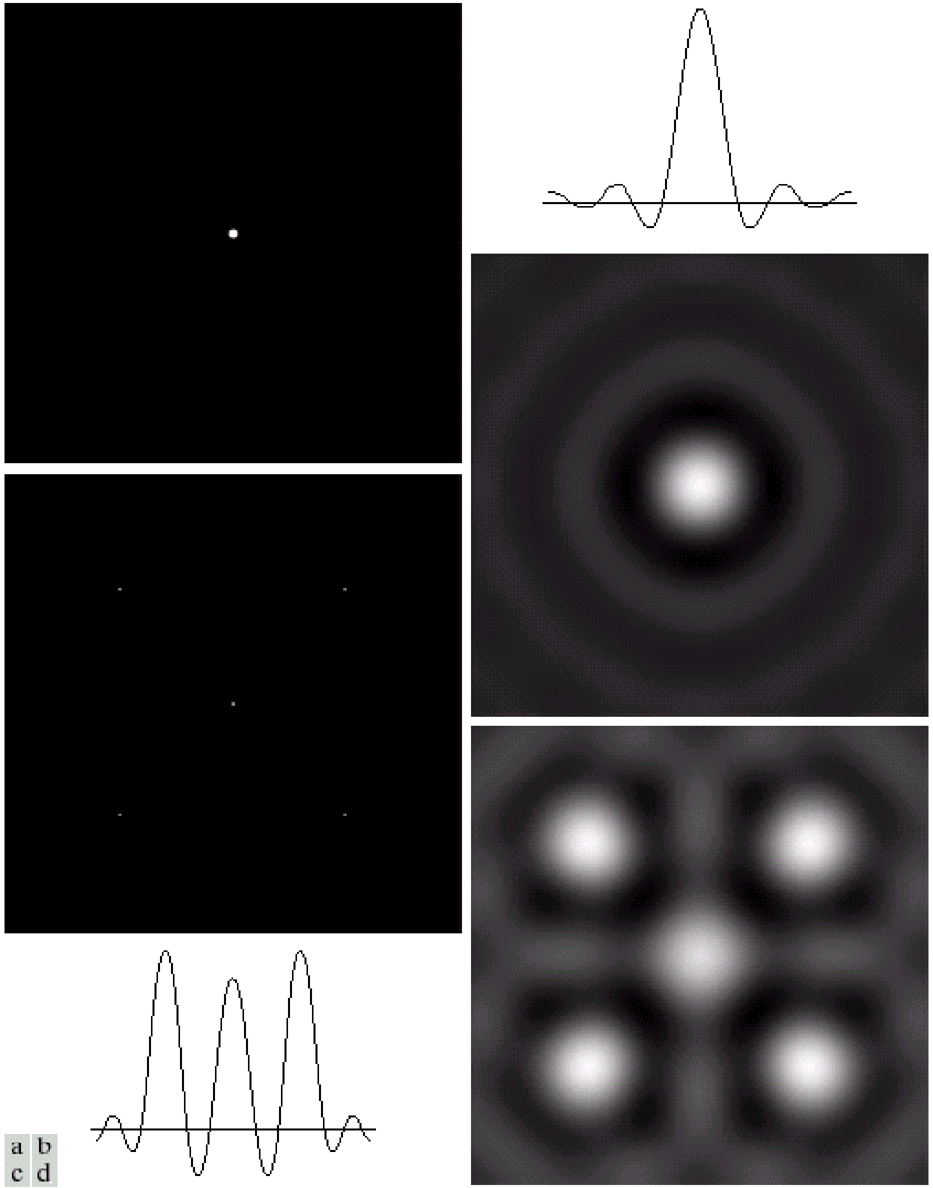


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

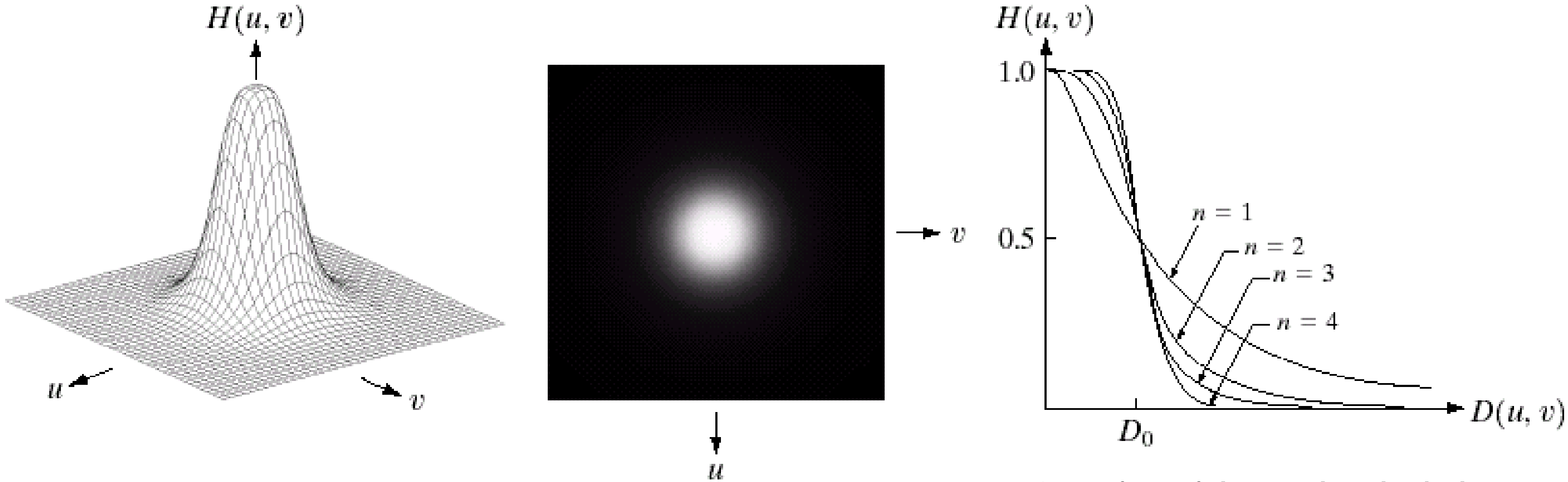
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Butterworth Lowpass Filter

Transfer function

$$H(u,v) = \frac{1}{1 + \left[D(u,v)/D_0\right]^{2N}}$$

Where D_0 = Cut off frequency, N = filter order.

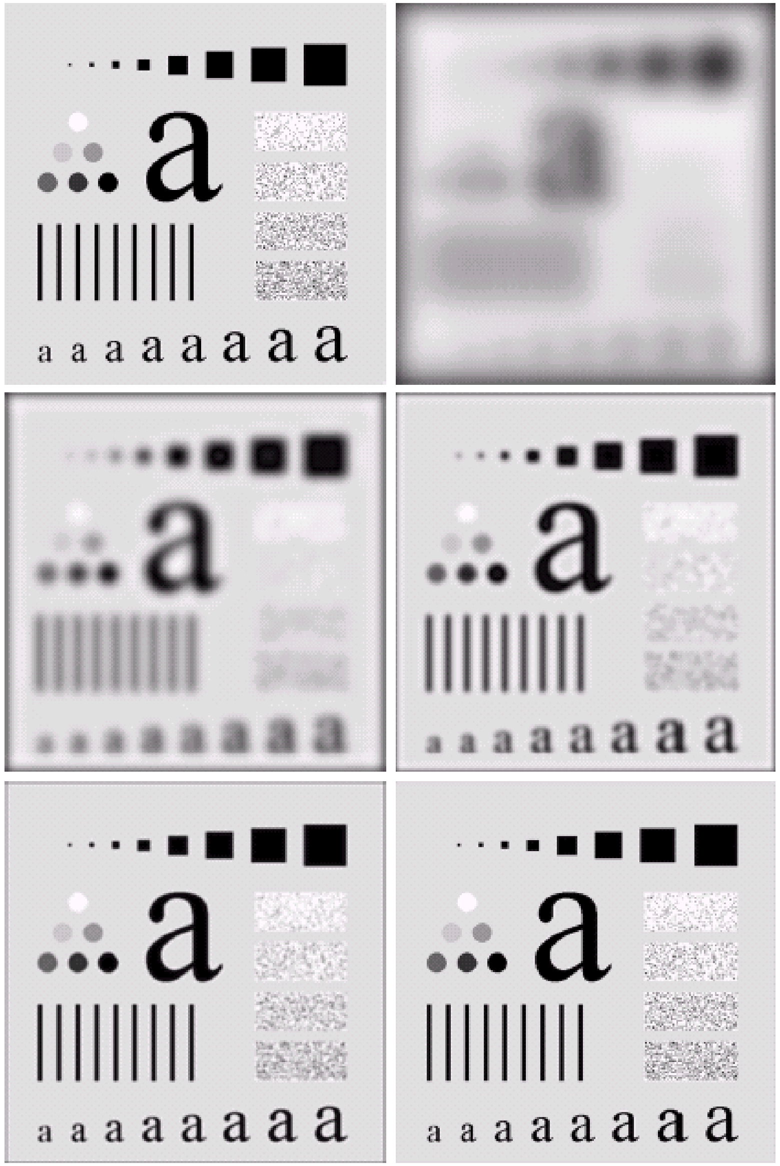


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Results of Butterworth Lowpass Filters



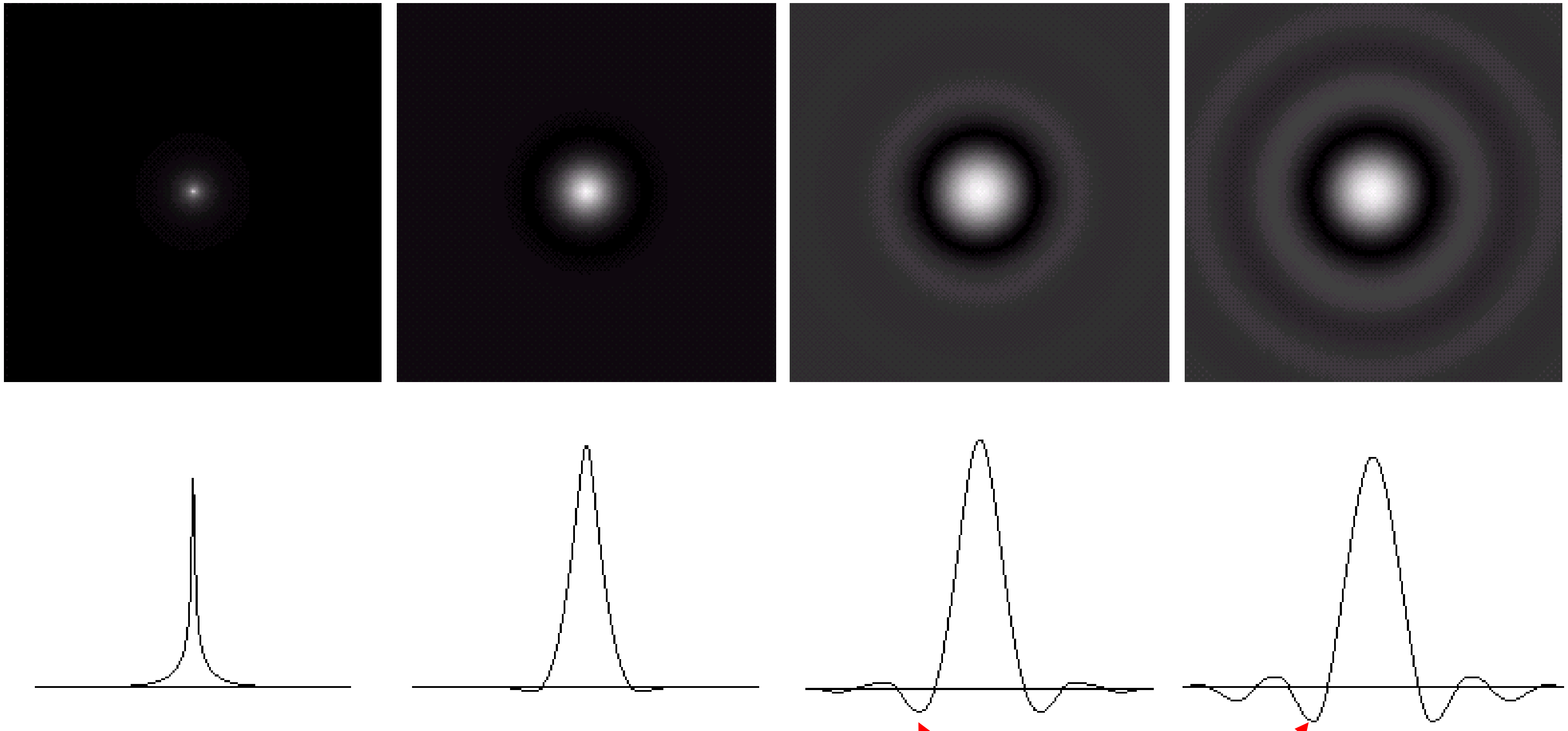
a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

There is less ringing effect compared to those of ideal lowpass filters!

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Spatial Masks of the Butterworth Lowpass Filters



a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Some ripples can be seen.

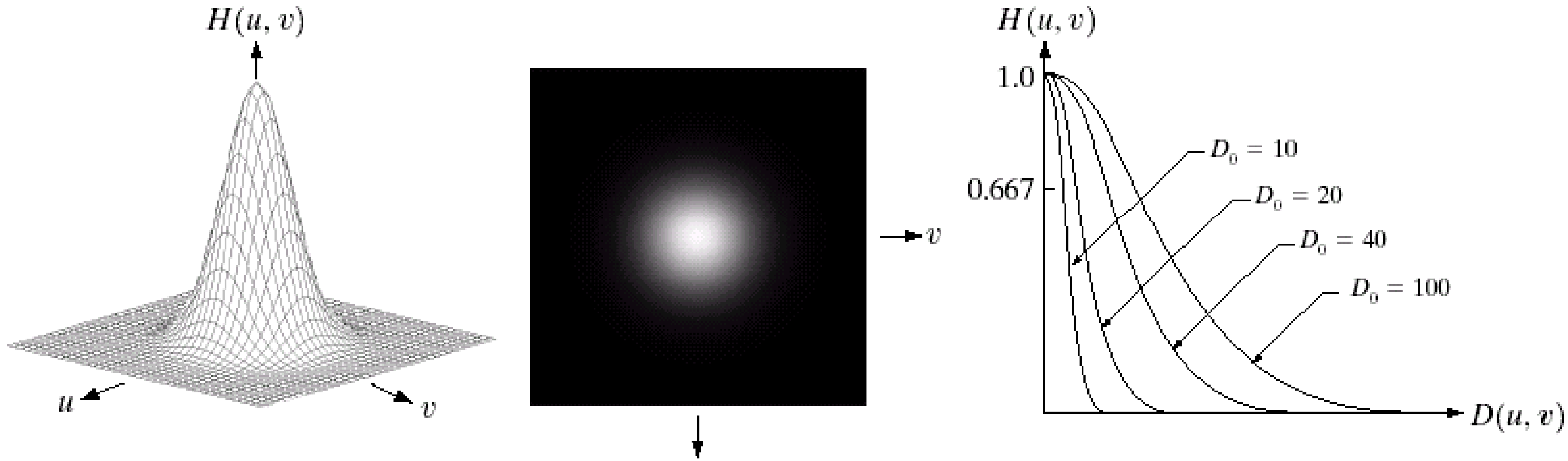
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gaussian Lowpass Filter

Transfer function

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Where D_0 = spread factor.



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

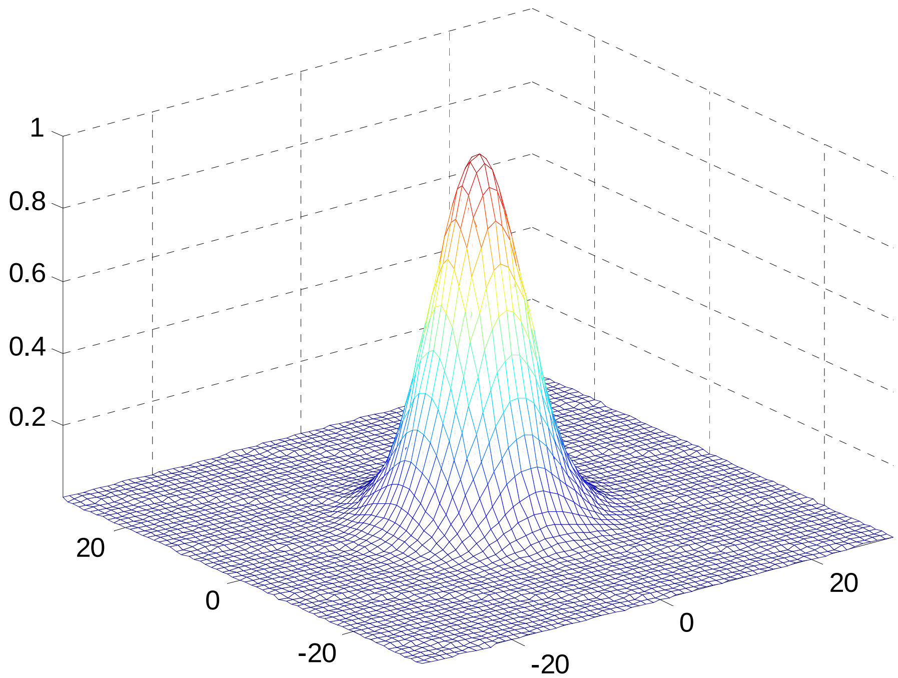
a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Note: the Gaussian filter is the only filter that has no ripple and hence no ringing effect.

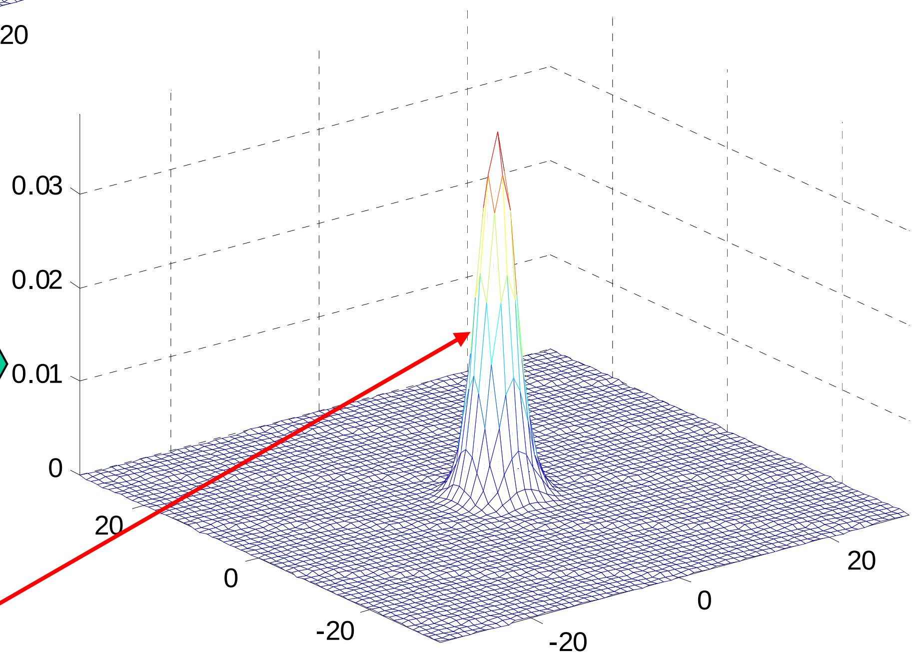
Gaussian Lowpass Filter (cont.)

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$



Gaussian lowpass filter with $D_0 = 5$

Spatial respones of the Gaussian lowpass filter with $D_0 = 5$



Gaussian shape

Results of Gaussian Lowpass Filters

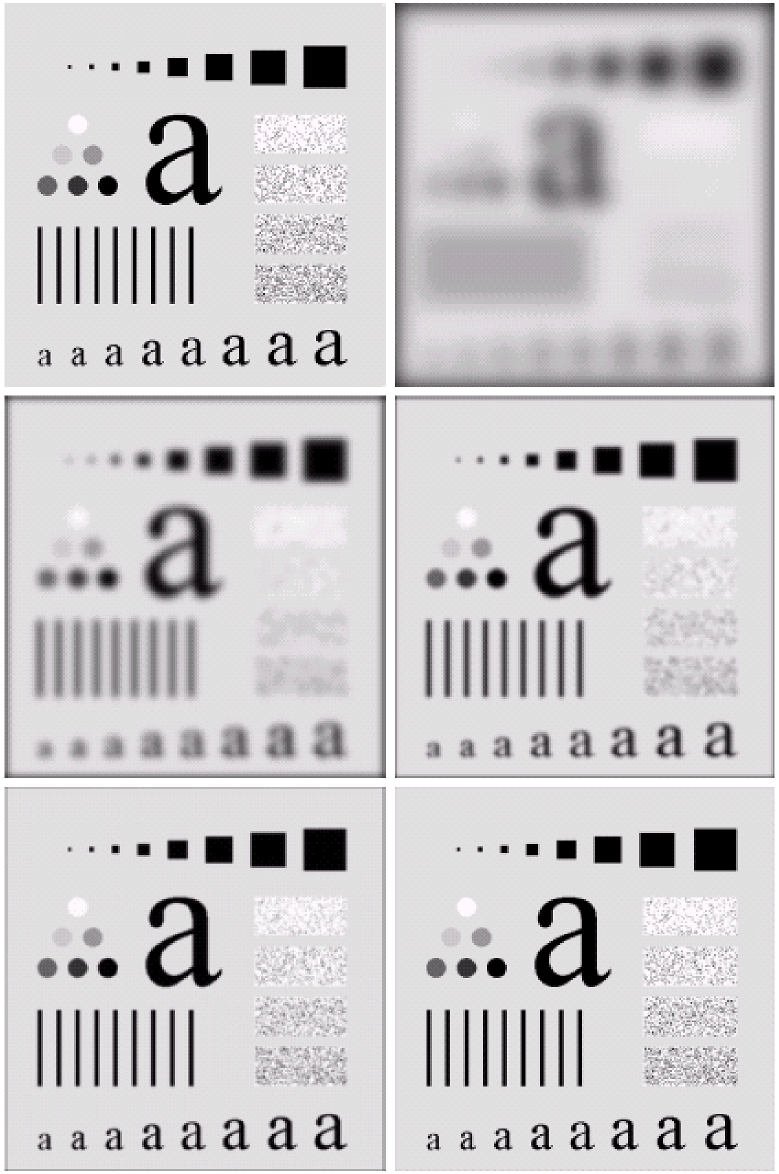


FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a	b
c	d
e	f

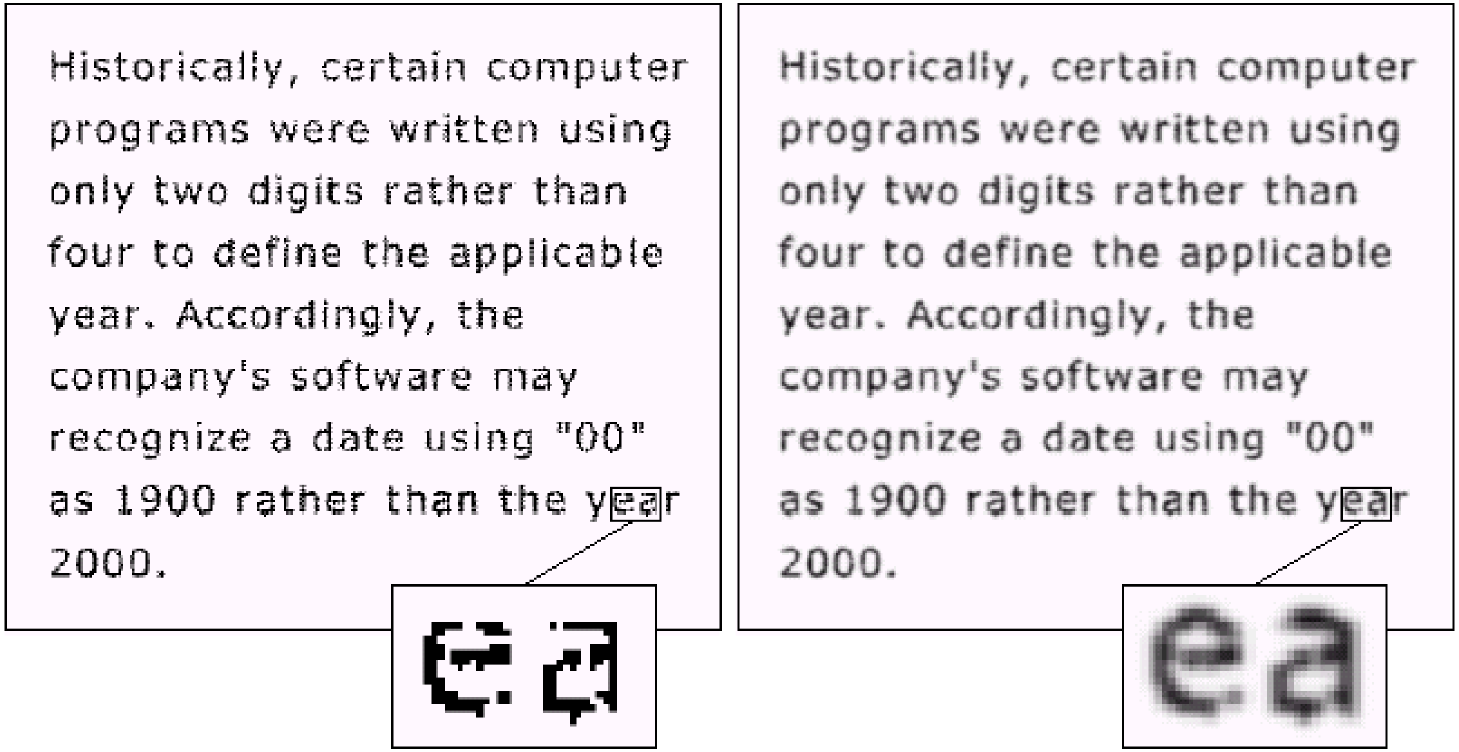
No ringing effect!

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Application of Gaussian Lowpass Filters

a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



Original image

Better Looking

The GLPF can be used to remove jagged edges and “repair” broken characters.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Application of Gaussian Lowpass Filters (cont.)

Remove wrinkles

Original image

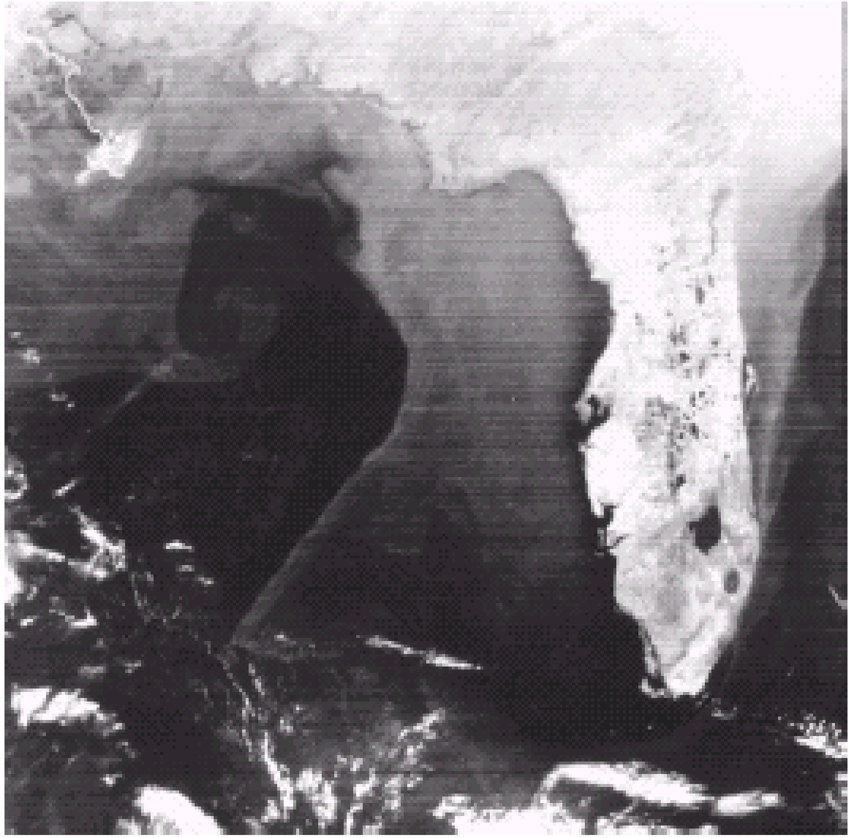
Softer-Looking

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Application of Gaussian Lowpass Filters (cont.)



Original image : The gulf of Mexico and Florida from NOAA satellite.

Filtered image

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Remove artifact lines: this is a simple but crude way to do it!

Highpass Filters

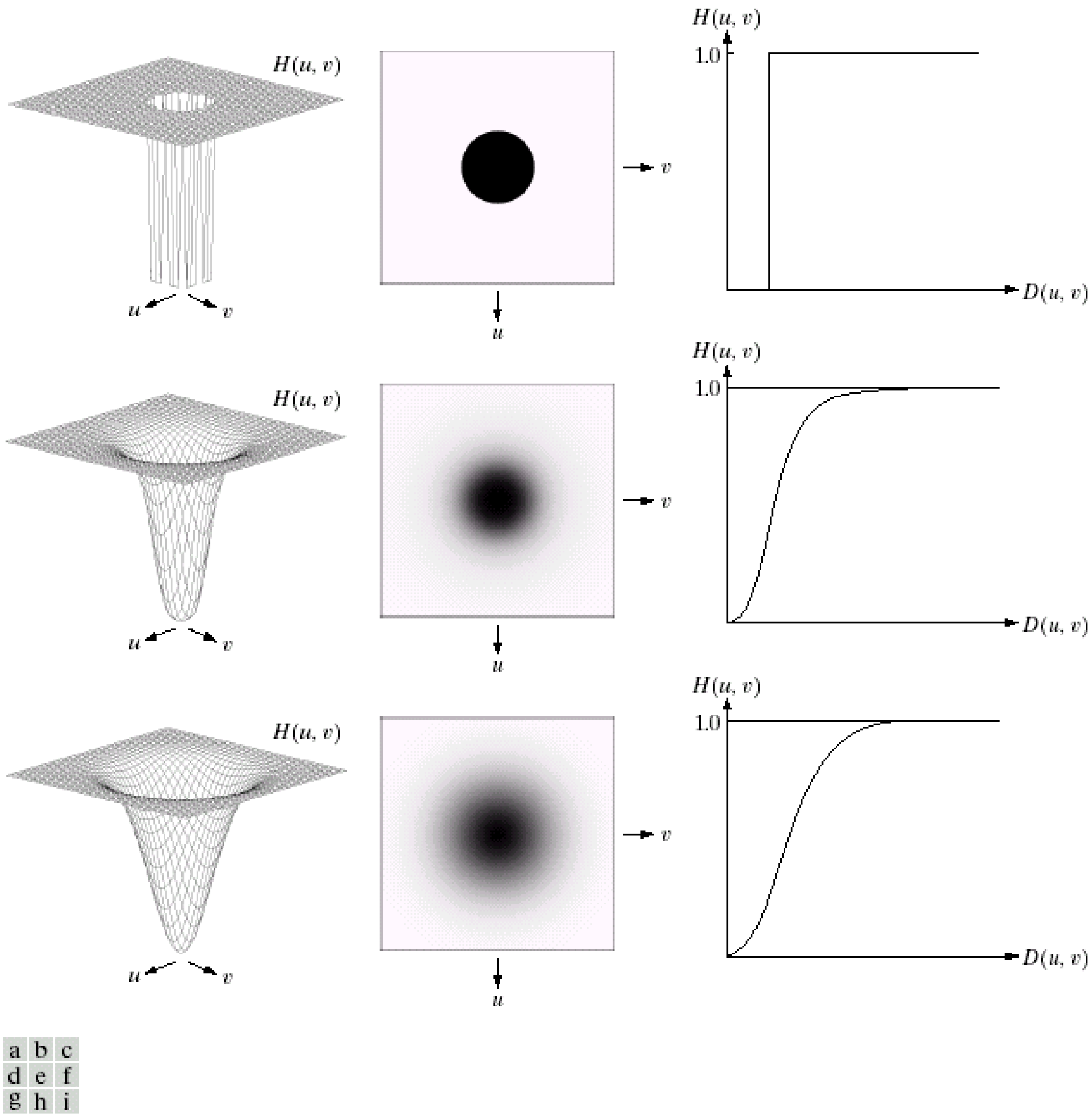


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

$$H_{hp} = 1 - H_{lp}$$

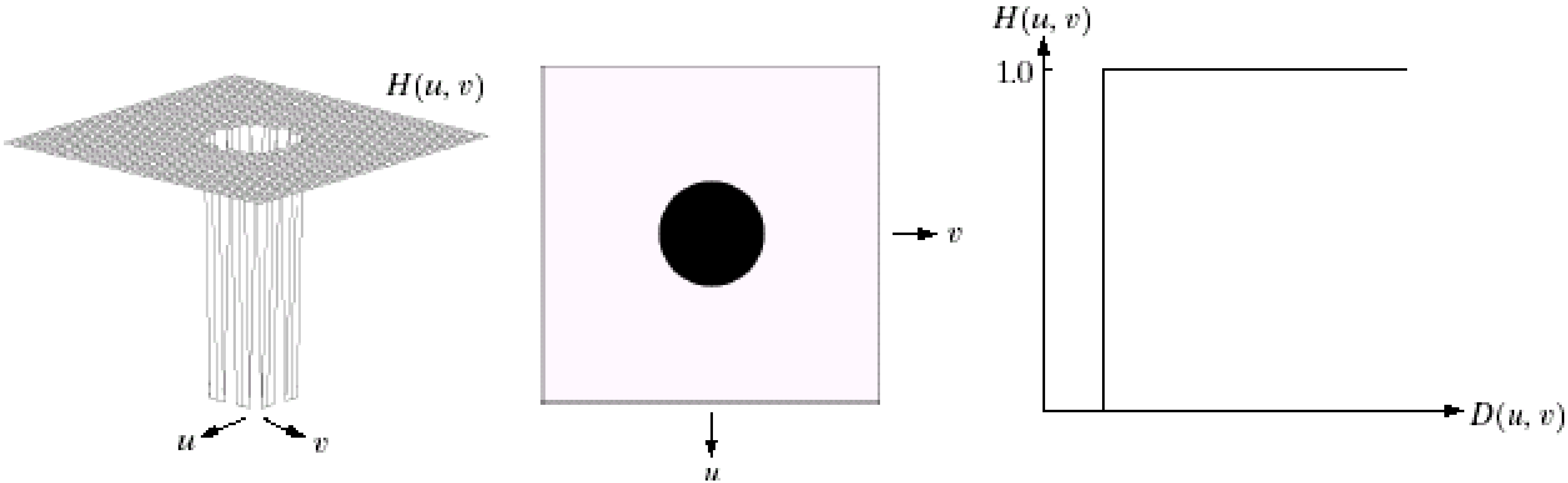
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Ideal Highpass Filters

Ideal LPF Filter Transfer function

$$H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 \\ 1 & D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ = Distance from (u,v) to the center of the mask.



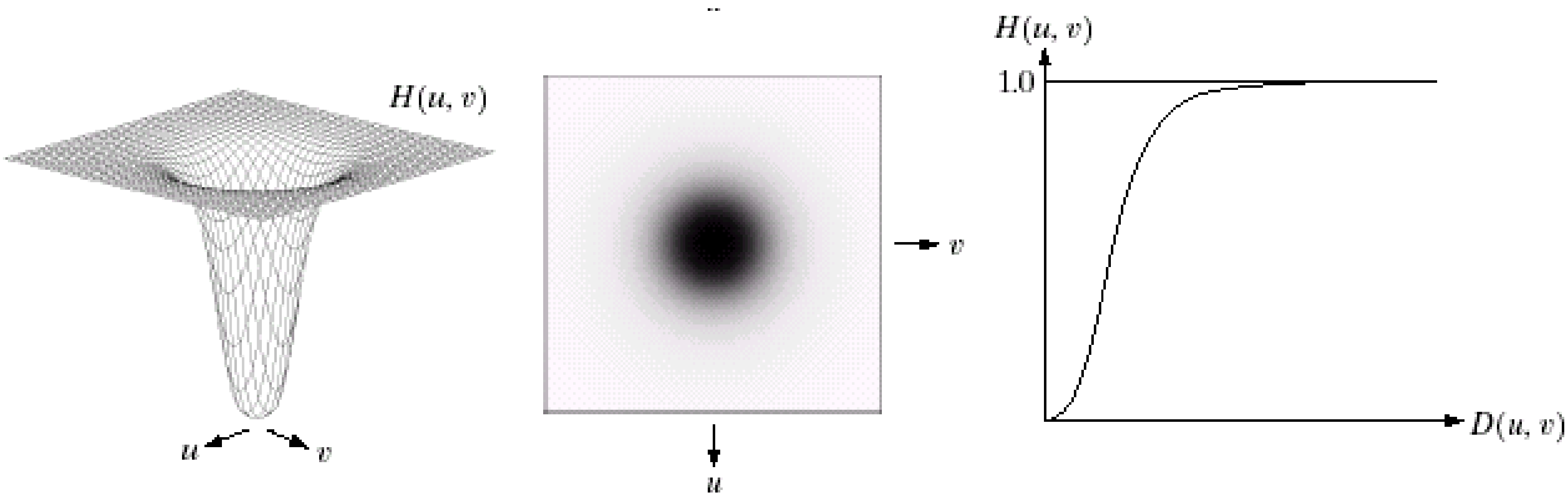
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Butterworth Highpass Filters

Transfer function

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v) \right]^{2N}}$$

Where D_0 = Cut off frequency, N = filter order.



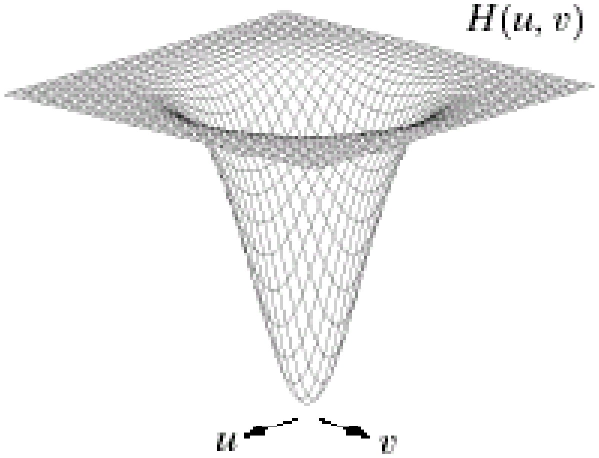
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

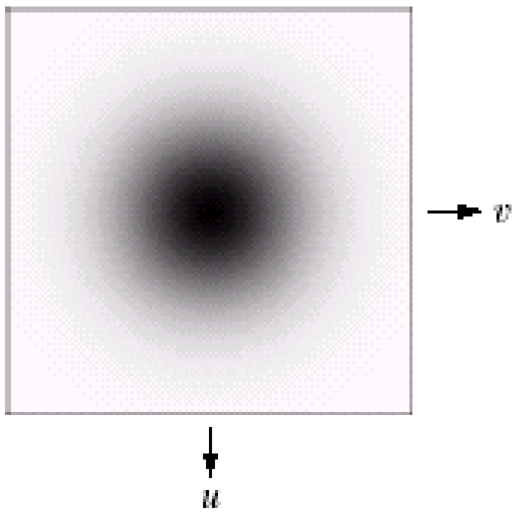
Gaussian Highpass Filters

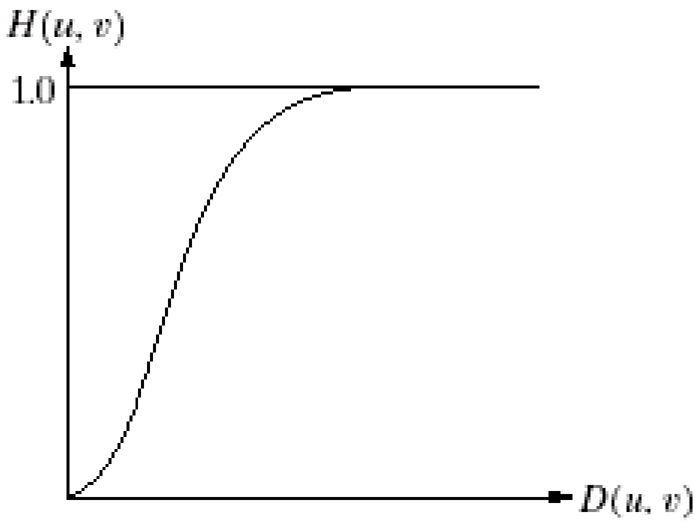
Transfer function

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Where D_0 = spread factor.

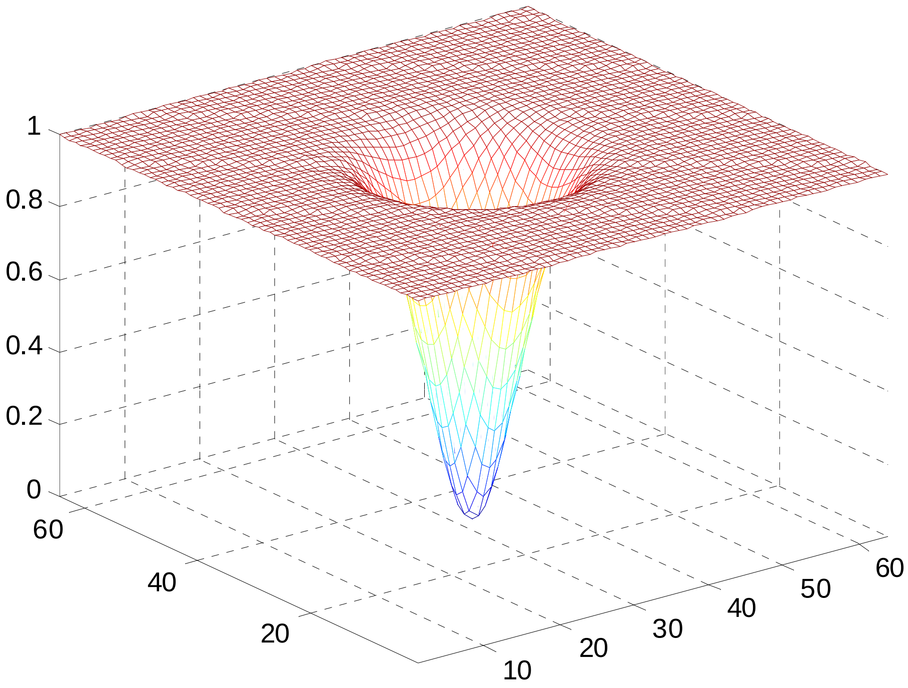






(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

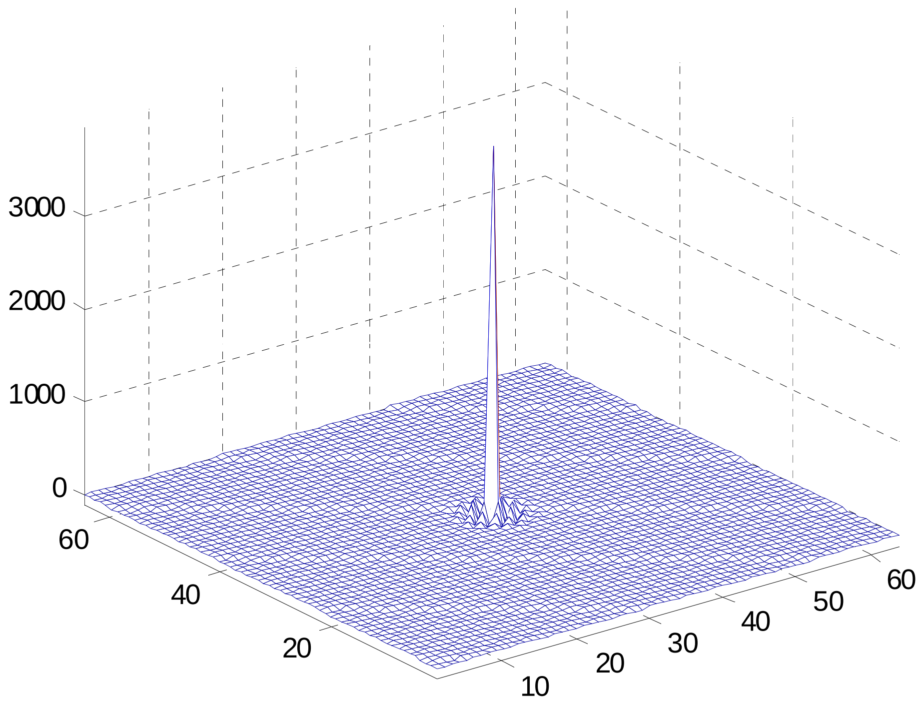
Gaussian Highpass Filters (cont.)



$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Gaussian highpass filter with $D_0 = 5$

Spatial respones of the Gaussian highpass filter with $D_0 = 5$



Spatial Responses of Highpass Filters

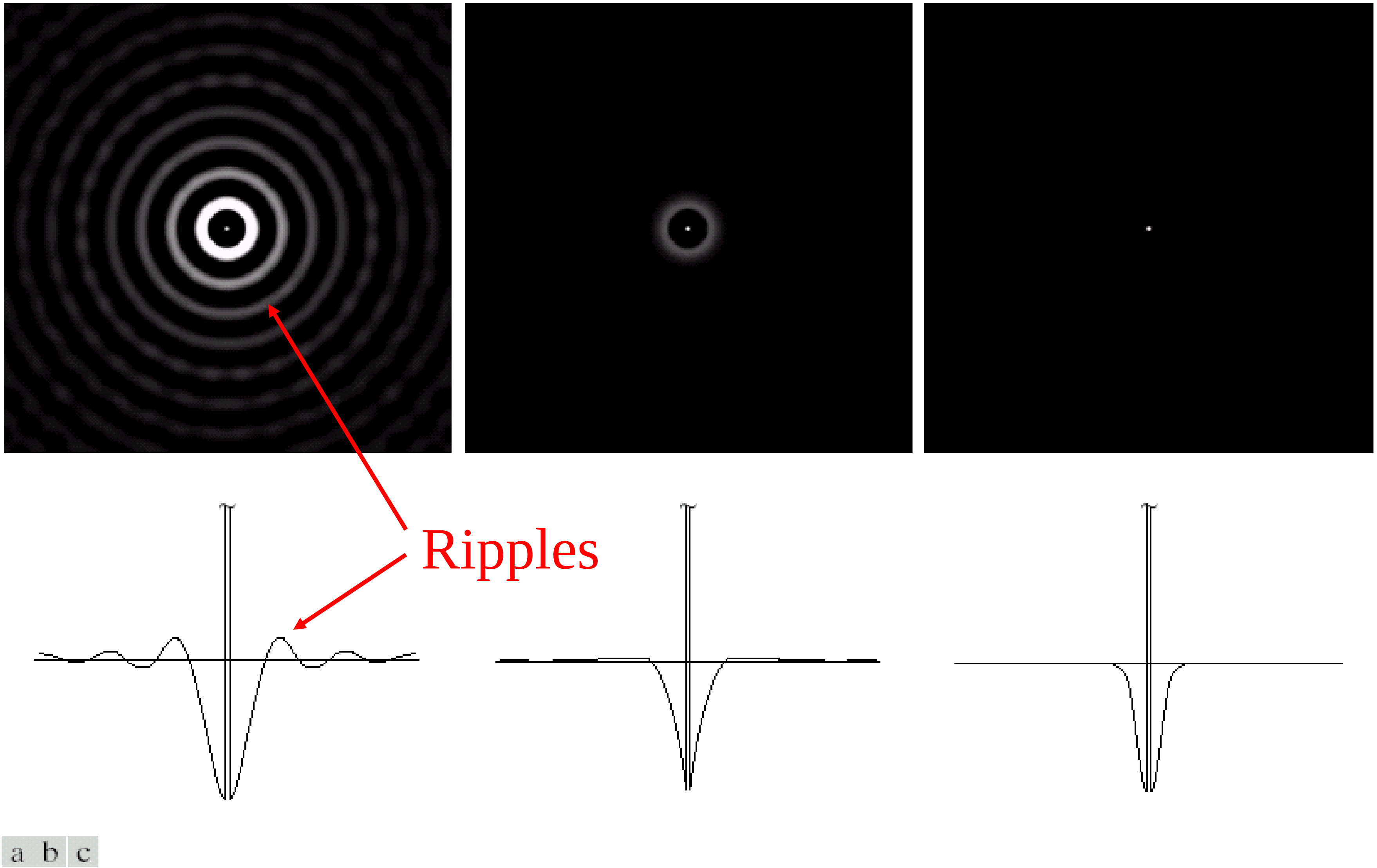
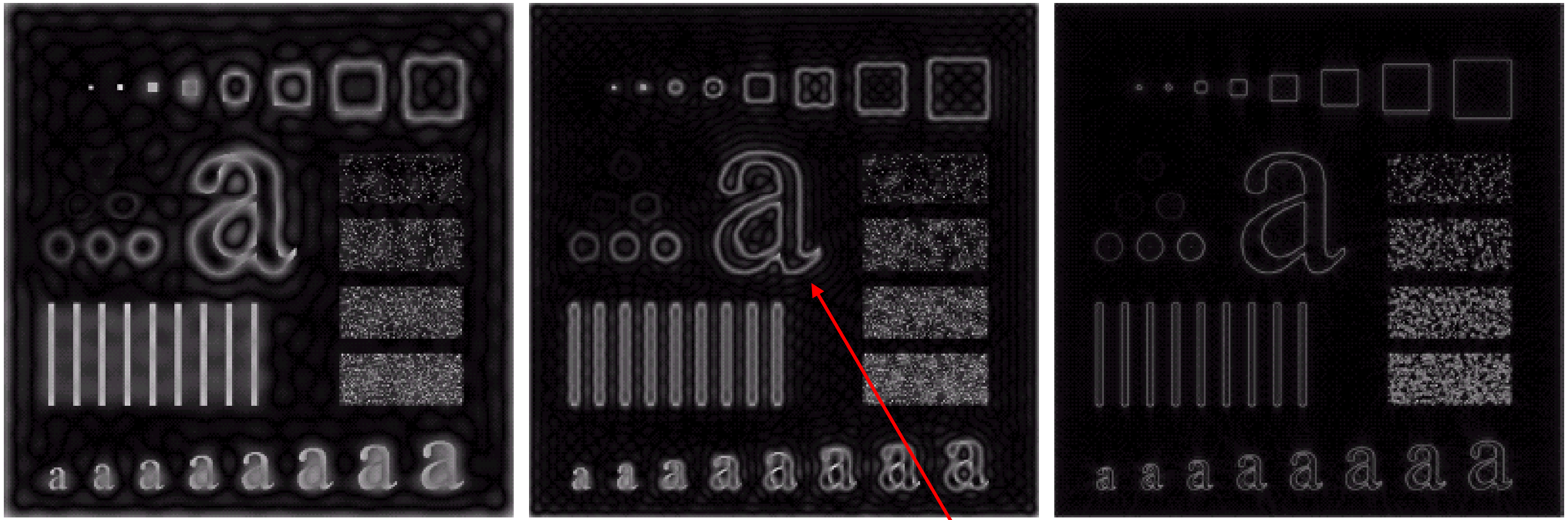


FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Results of Ideal Highpass Filters



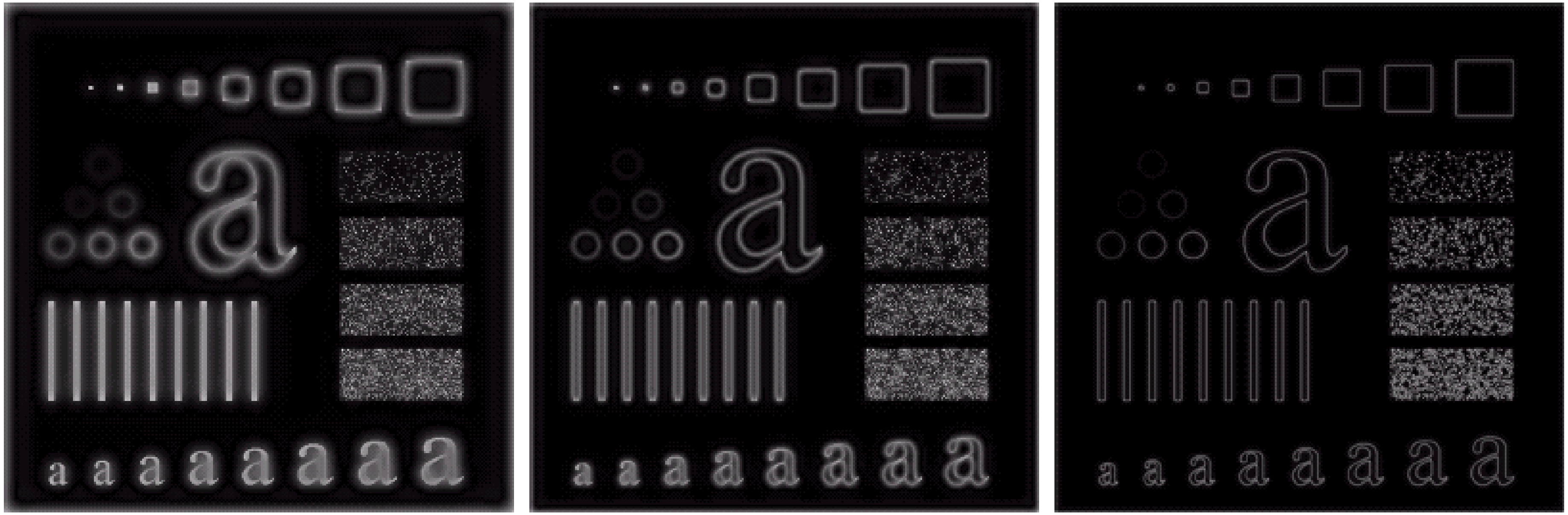
a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

Ringing effect can be obviously seen!

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Results of Butterworth Highpass Filters

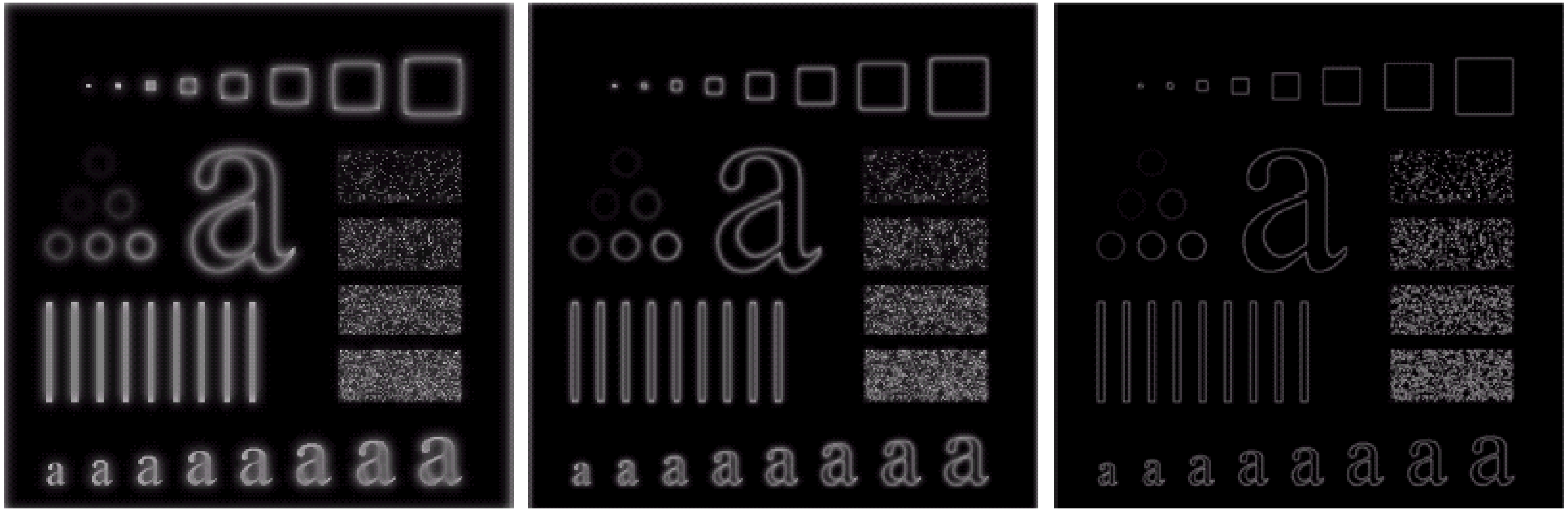


a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Results of Gaussian Highpass Filters



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Laplacian Filter in the Frequency Domain

From Fourier Tr. Property:

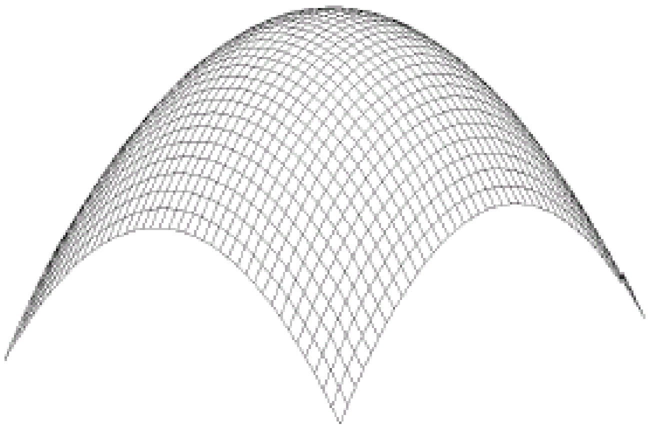
$$\frac{d^n f(x)}{dx^n} \Leftrightarrow (ju)^n F(u)$$

Then for Laplacian operator

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \Leftrightarrow -\left(u^2 + v^2\right) F(u,v)$$

We get

$$\nabla^2 \Leftrightarrow -\left(u^2 + v^2\right)$$



Surface plot

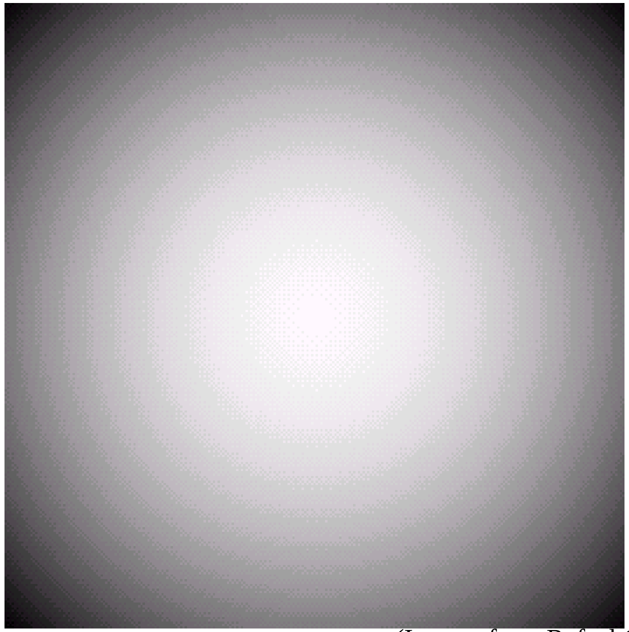
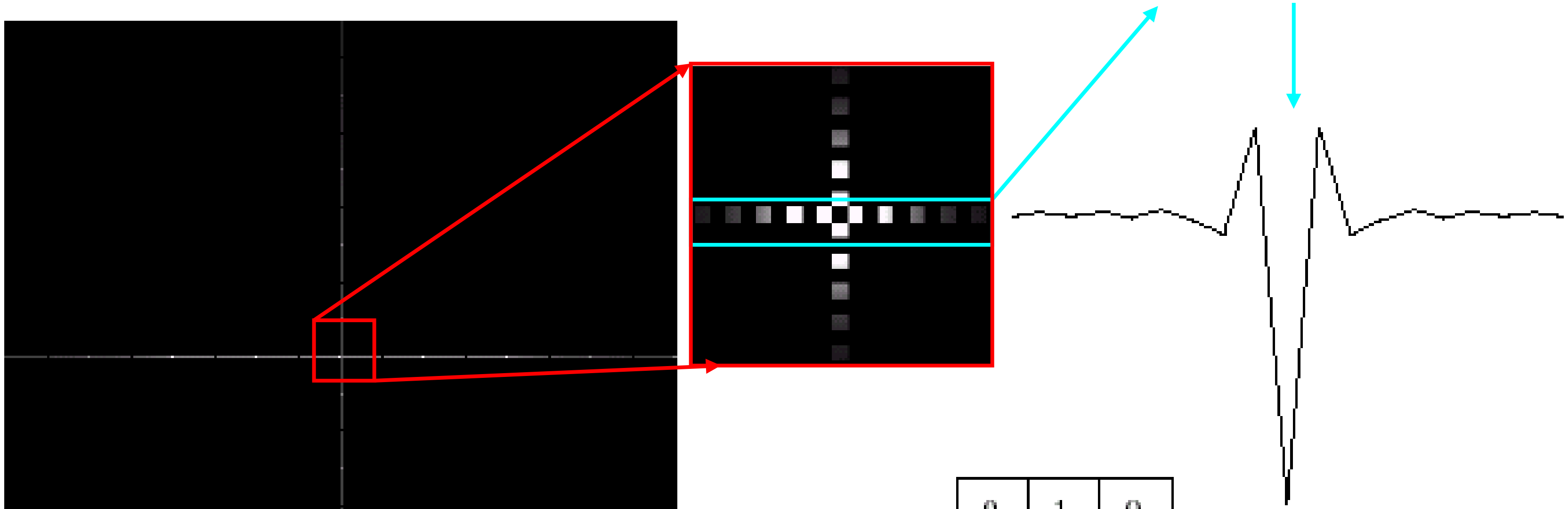


Image of $-(u^2+v^2)$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Laplacian Filter in the Frequency Domain (cont.)

Spatial response of $-(u^2+v^2)$



0	1	0
1	–4	1
0	1	0

Laplacian mask in Chapter 3

Sharpening Filtering in the Frequency Domain

Spatial Domain

$$f_{hp}(x,y) = f(x,y) - f_{lp}(x,y)$$

$$f_{hb}(x,y) = Af(x,y) - f_{lp}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - f_{lp}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y)$$

Frequency Domain Filter

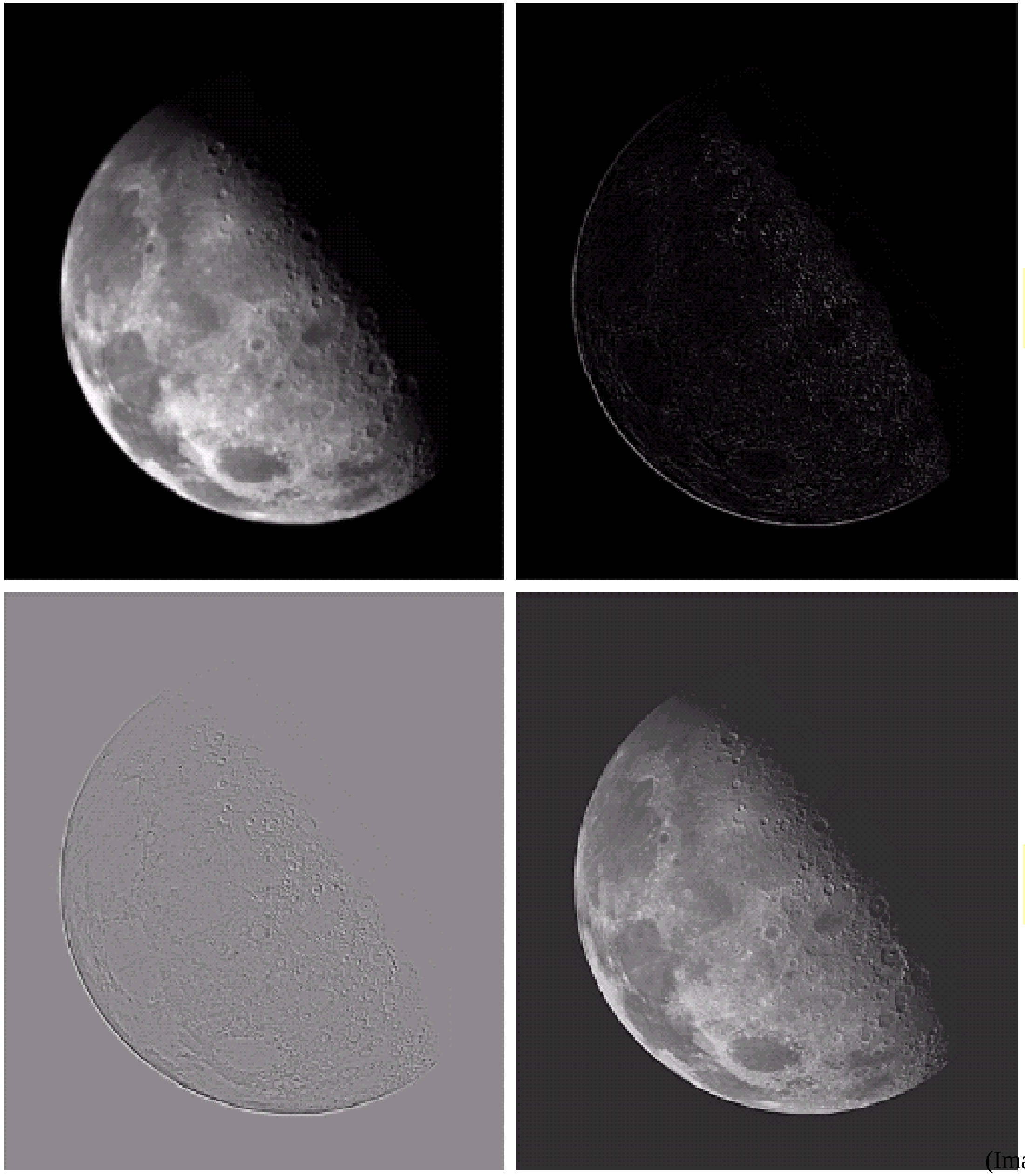
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

$$H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Sharpening Filtering in the Frequency Domain (cont.)

p $\nabla^2 P$



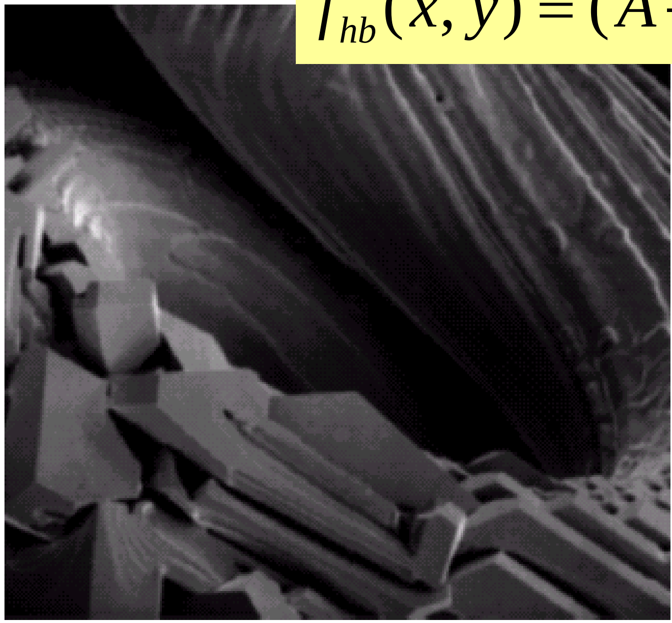
$\nabla^2 P$ $P - \nabla^2 P$

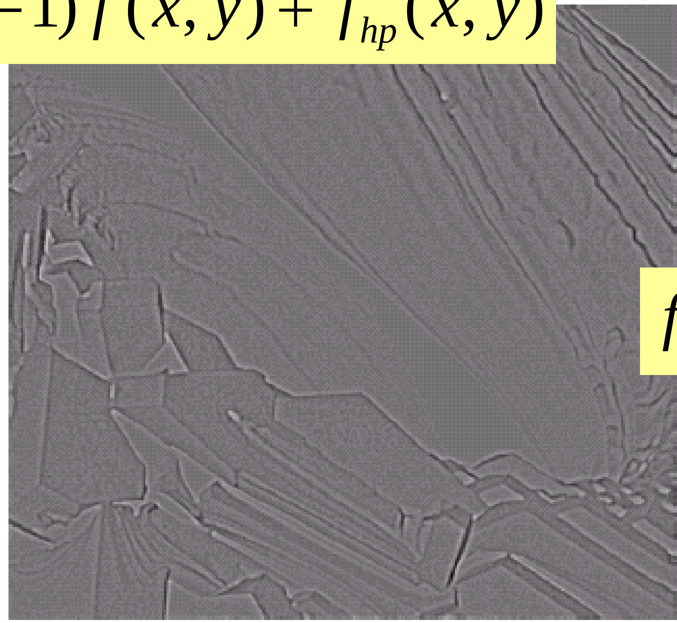
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Sharpening Filtering in the Frequency Domain (cont.)

$$f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y)$$

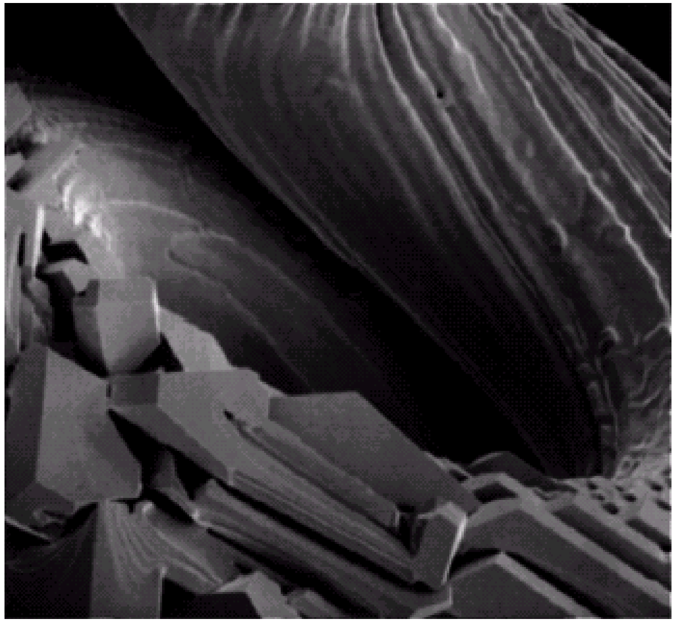
f

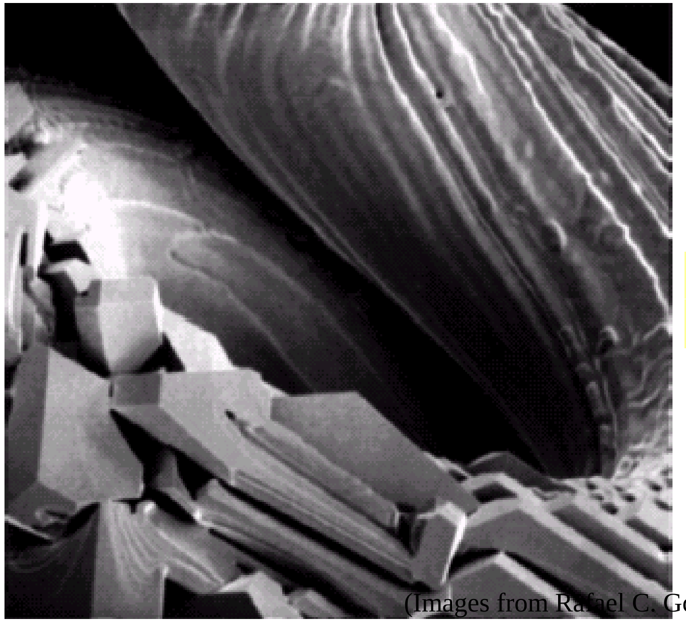




$$f_{hp} = \nabla^2 P$$

$A = 2$



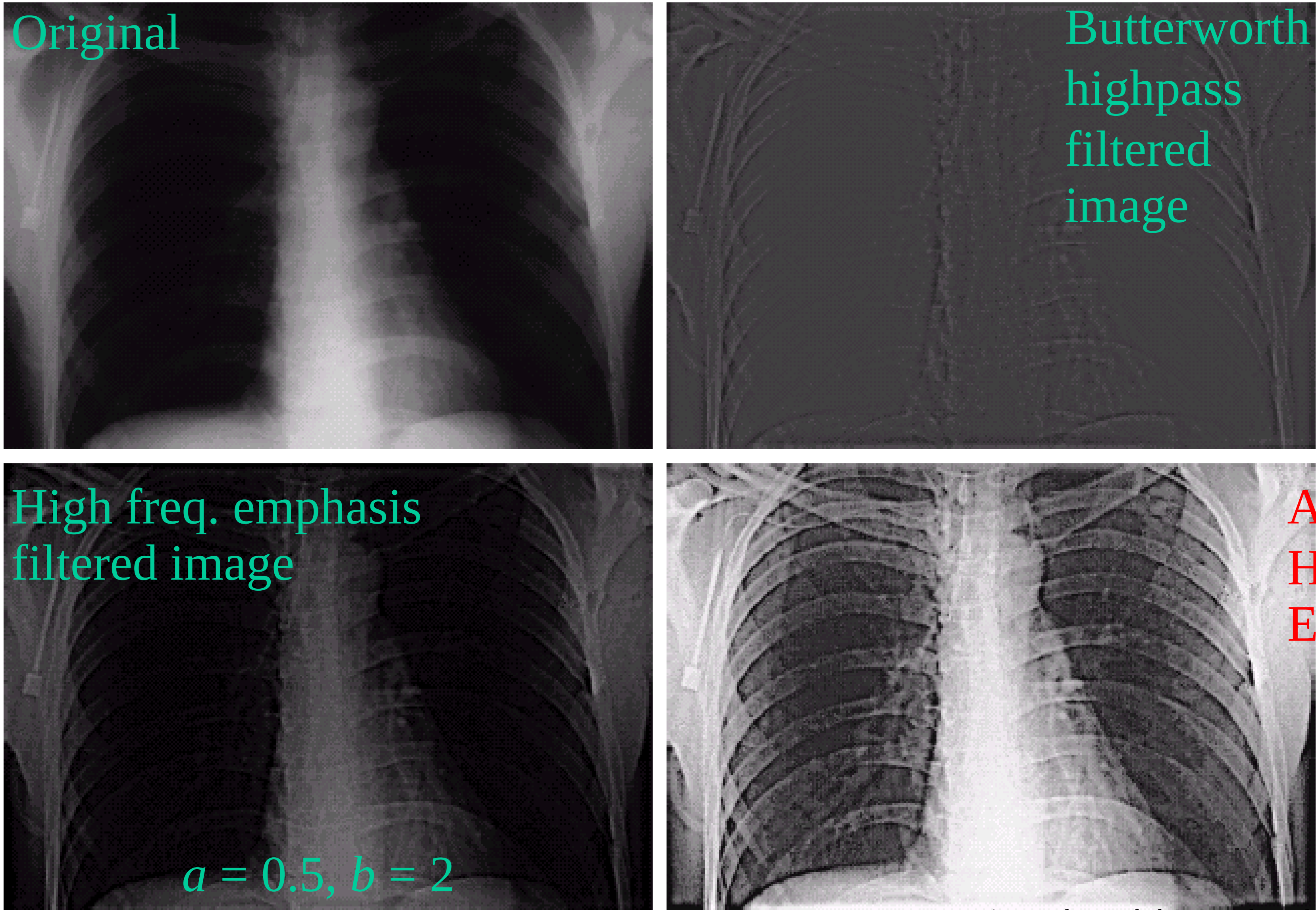


$A = 2.7$

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

High Frequency Emphasis Filtering

$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Homomorphic Filtering

An image can be expressed as

$$f(x, y) = i(x, y)r(x, y)$$

$i(x,y)$ = illumination component
$r(x,y)$ = reflectance component

We need to suppress effect of illumination that cause image Intensity changed slowly.

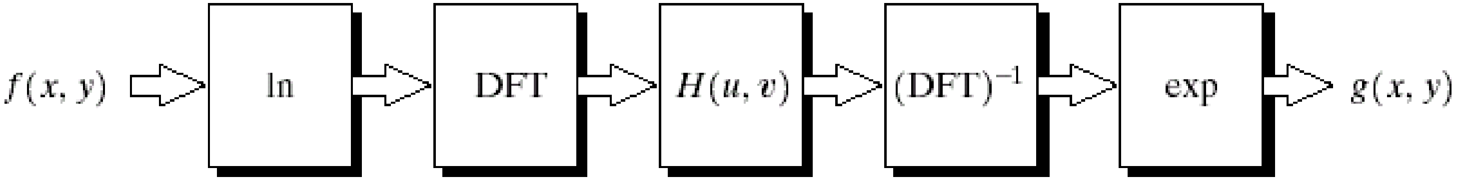


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

Homomorphic Filtering

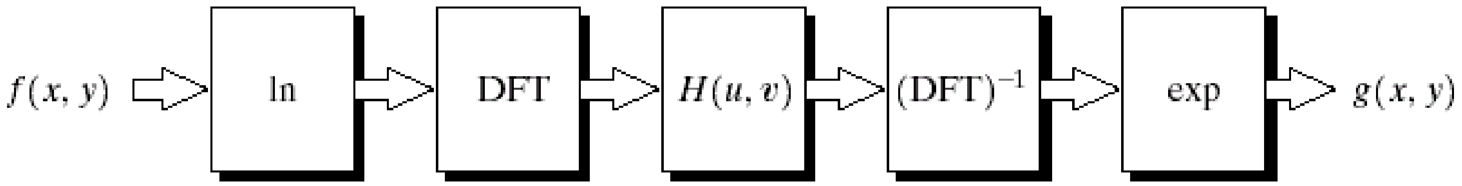


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

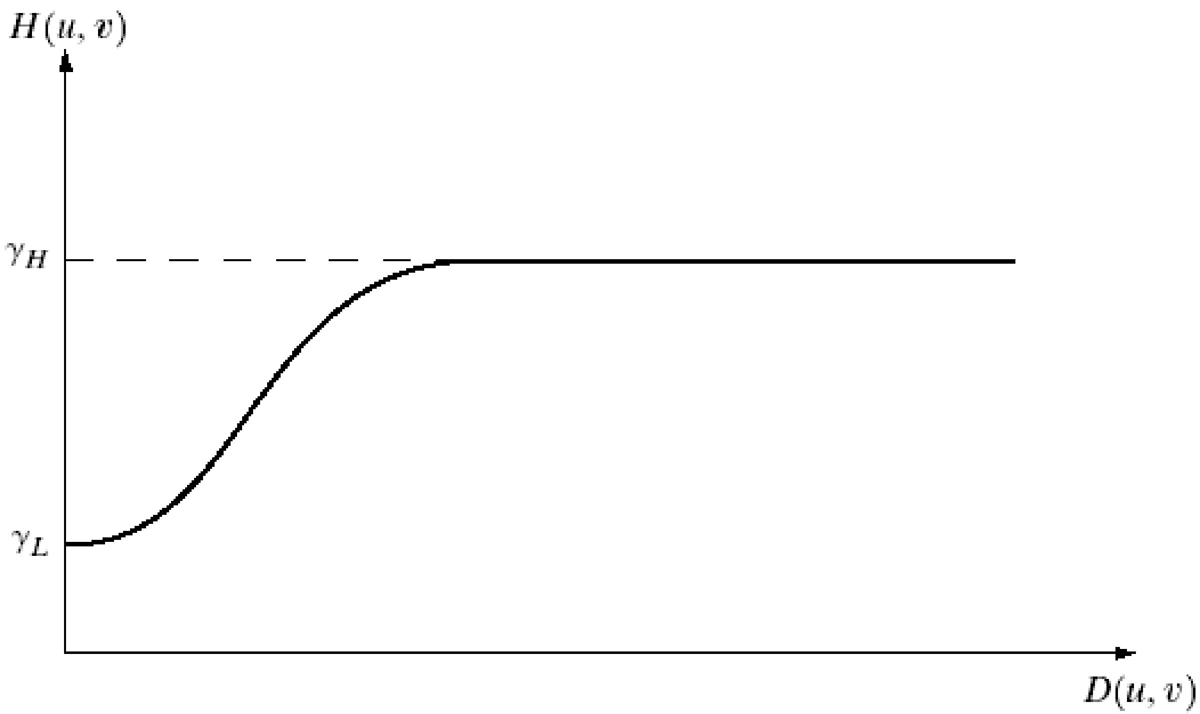
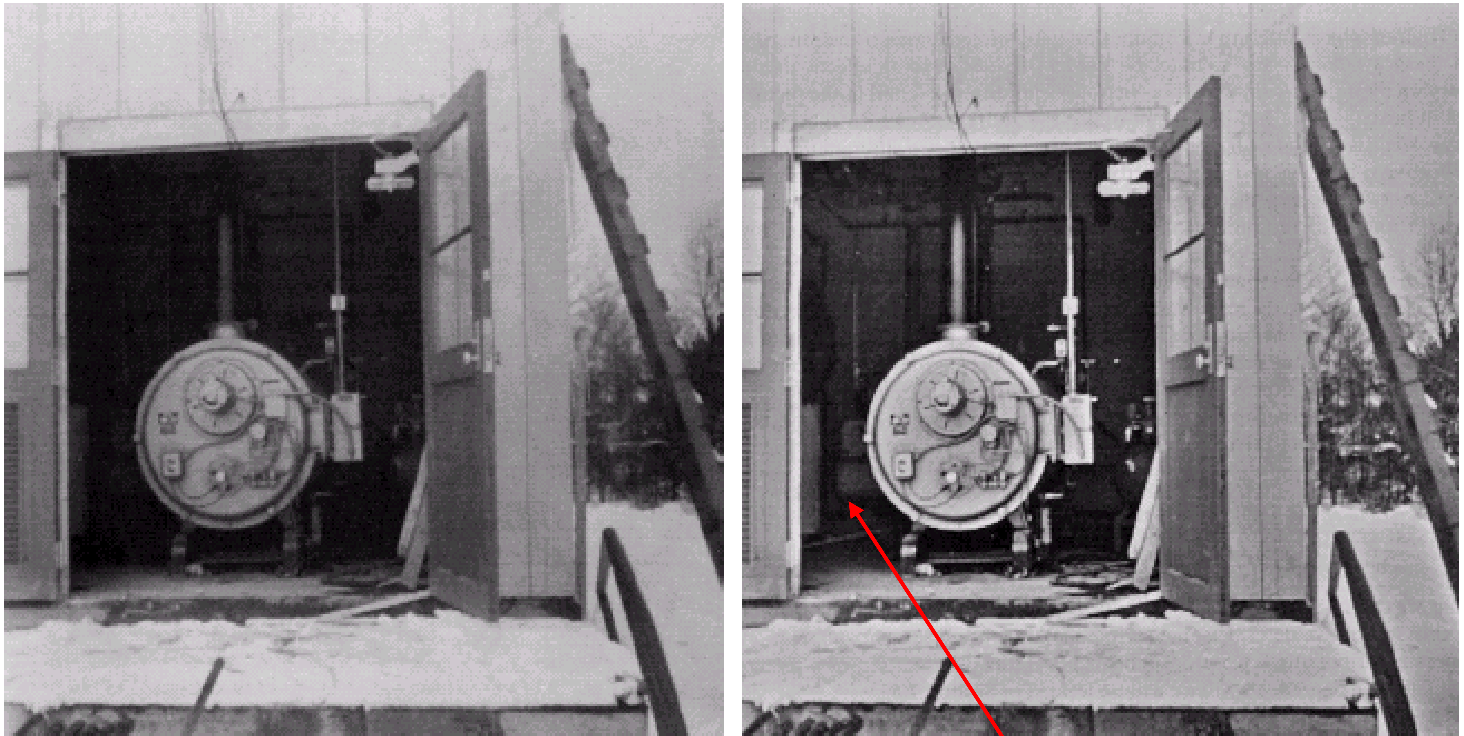


FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Homomorphic Filtering



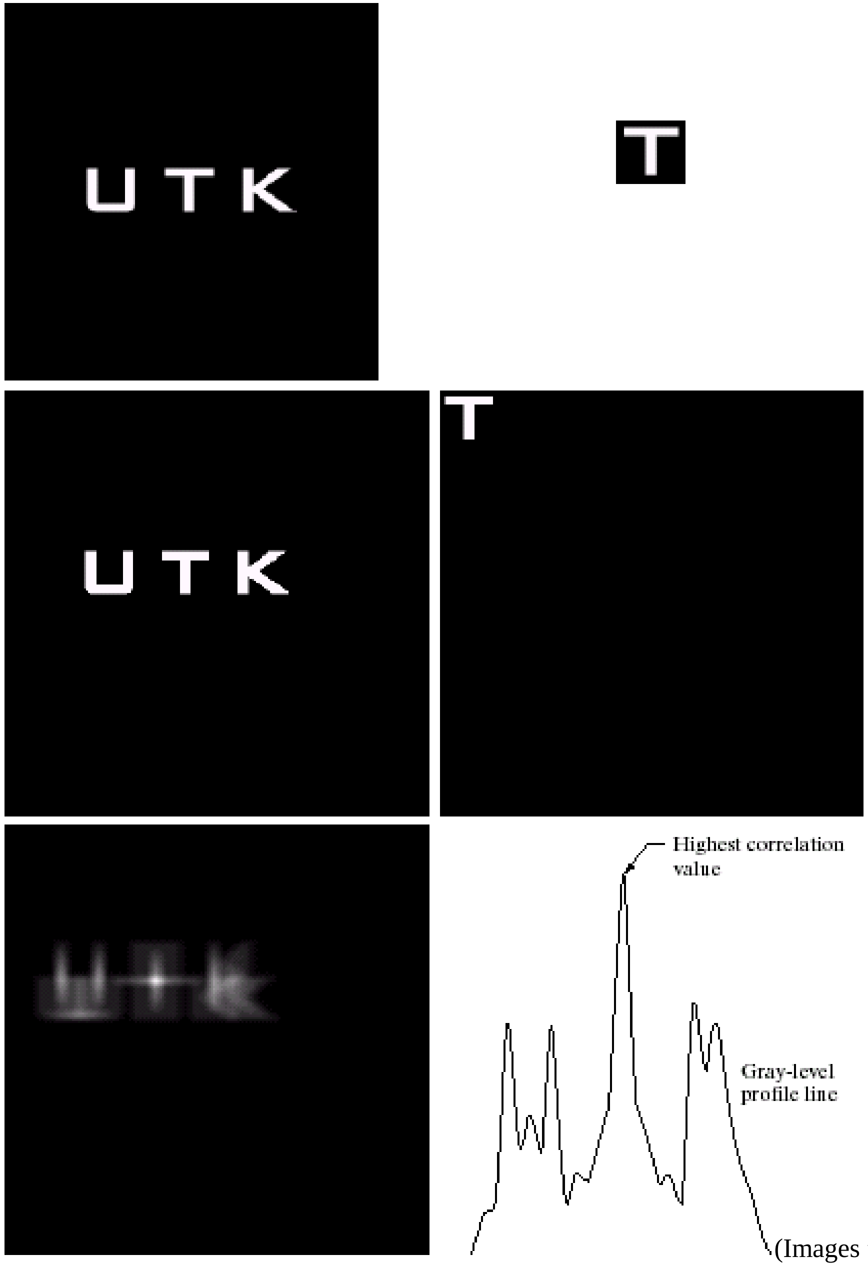
a b

FIGURE 4.33
(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

More details in the room can be seen!

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Correlation Application: Object Detection



a b
c d
e f

FIGURE 4.41 (a) Image. (b) Template. (c) and (d) Padded images. (e) Correlation function displayed as an image. (f) Horizontal profile line through the highest value in (e), showing the point at which the best match took place.

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